

thm_2Ereal__topology_2ELIFT__TO__QUOTIENT__SPACE (TMZzB7Y2VofxGCtYNUdiuJa8ZNqJBhH6gRQ)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 5 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.\lambda V0f \in (A.\lambda 27b^{A-27c}).\lambda V1g$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (1)$$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty\ A.\lambda 27b \Rightarrow c_2Epair_2EABS_prod\ A.\lambda 27a\ A.\lambda 27b \in ((ty_2Epair_2Eprod\ A.\lambda 27a\ A.\lambda 27b)^{(2^{A-27b})^{A-27a}}) \quad (3)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda V0x \in A.\lambda 27a.\lambda V1y \in A.\lambda 27b.(ap (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0a)))$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 10 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 13 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal\ V0P))))$

Definition 14 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

Definition 16 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in ((2^{(2^A-27a)})^{(ty_2Etopology_2Etopology A_27a)}) \quad (15)$$

Definition 17 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c$

Definition 18 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 19 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1s \in$

Definition 20 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap ($

Definition 21 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 22 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E21 2) (\lambda V2t \in$

Definition 23 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).(((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (29)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (30)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q) \Rightarrow (p V0r)) \Leftrightarrow ((p V2q) \Rightarrow ((p V1p) \Rightarrow (p V0r)))))) \Rightarrow (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27c}^{A_{.27a}}).(\forall V2x \in A_{.27c}.((ap (ap (ap (c_{.2E}combin_{.2E}o A_{.27c} A_{.27b} A_{.27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \Rightarrow (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V1t)))))) \Rightarrow (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p (ap (ap (c_{.2E}pred_{.set}_{.2E}SUBSET A_{.27a}) V0s) V0s))) \Rightarrow (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0y \in A_{.27b}.(\forall V1s \in (2^{A_{.27a}}).(\forall V2f \in (A_{.27b}^{A_{.27a}}).((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27b}) V0y) (ap (ap (c_{.2E}pred_{.set}_{.2E}IMAGE A_{.27a} A_{.27b}) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_{.27a}.((V0y = (ap V2f V3x)) \wedge (p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V3x) V1s)))))) \Rightarrow (35)$$

Assume the following.

$$(\forall V0f \in (ty_{.2E}realax_{.2E}real^{ty_{.2E}realax_{.2E}real}).(\forall V1g \in (ty_{.2E}realax_{.2E}real^{ty_{.2E}realax_{.2E}real}).(\forall V2s \in (2^{ty_{.2E}realax_{.2E}real}).(((\forall V3x \in ty_{.2E}realax_{.2E}real.((p (ap (ap (c_{.2E}bool_{.2E}IN ty_{.2E}realax_{.2E}real) V3x) V2s)) \Rightarrow ((ap V0f V3x) = (ap V1g V3x)))) \wedge (p (ap (ap c_{.2E}real_{.topology}_{.2E}continuous_{.on} V0f) V2s))) \Rightarrow (p (ap (ap c_{.2E}real_{.topology}_{.2E}continuous_{.on} V1g) V2s)))))) \Rightarrow (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3t \in (2^{ty_2Erealax_2Ereal}).(\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V0f) V2s)) V3t)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V1g) V3t)) V4u)) \wedge ((\forall V5v \in (2^{ty_2Erealax_2Ereal}).((p (\\
& ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V5v) V3t)) \Rightarrow \\
& ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
c_2Ereal_topology_2Eeuclidean) V2s)) (ap (c_2Epred_set_2EGSPEC \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V6x \in ty_2Erealax_2Ereal. \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V6x) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x) V2s)) (ap (ap (\\
& c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V6x)) V5v)))))) \Leftrightarrow (\\
& p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap \\
& (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) \\
& V3t)) V5v)))) \wedge (p (ap (ap (c_2Ereal_topology_2Econtinuous_on \\
& (ap (ap (c_2Ecombin_2Eo ty_2Erealax_2Ereal ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V1g) V0f)) V2s)))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V1g) V3t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c. \\
& nonempty A.27c \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1f \in (A.27b^{A.27a}). \\
& (\forall V2g \in (A.27c^{A.27a}).((\forall V3x \in A.27a.(\forall V4y \in \\
& A.27a.(((p (ap V0P V3x)) \wedge ((p (ap V0P V4y)) \wedge ((ap V2g V3x) = (ap V2g \\
& V4y)))) \Rightarrow ((ap V1f V3x) = (ap V1f V4y)))))) \Leftrightarrow (\exists V5h \in (A.27b^{A.27c}). \\
& (\forall V6x \in A.27a.((p (ap V0P V6x)) \Rightarrow ((ap V1f V6x) = (ap V5h (ap V2g \\
& V6x))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (53)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1h \in \\ & (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\ & (\forall V3t \in (2^{ty_2Erealax_2Ereal}).(\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\ & (((ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V0f) V2s) = V3t) \wedge ((\forall V5v \in (2^{ty_2Erealax_2Ereal}).((p (ap \\ & (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V5v) V3t)) \Rightarrow \\ & ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\ & (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\ & c_2Ereal_topology_2Euclidean) V2s)) (ap (c_2Epred_set_2EGSPEC \\ & ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V6x \in ty_2Erealax_2Ereal. \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V6x) (ap (ap c_2Ebool_2E_2F_5C \\ & (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x) V2s)) (ap (ap (\\ & c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V6x)) V5v)))))) \Leftrightarrow (\\ & p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap \\ & (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) \\ & V3t)) V5v)))))) \wedge ((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\ & V1h) V2s)) \wedge (((ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V1h) V2s) = V4u) \wedge (\forall V7x \in ty_2Erealax_2Ereal. \\ & (\forall V8y \in ty_2Erealax_2Ereal.(((p (ap (ap (c_2Ebool_2EIN \\ & ty_2Erealax_2Ereal) V7x) V2s)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V8y) V2s)) \wedge ((ap V0f V7x) = (ap V0f V8y)))))) \Rightarrow ((ap V1h V7x) = (ap V1h V8y))))))))) \Rightarrow \\ & (\exists V9g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).((p (\\ & ap (ap c_2Ereal_topology_2Econtinuous_on V9g) V3t)) \wedge ((ap \\ & (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V9g) V3t) = V4u) \wedge (\forall V10x \in ty_2Erealax_2Ereal.((p (ap (ap \\ & (c_2Ebool_2EIN ty_2Erealax_2Ereal) V10x) V2s)) \Rightarrow ((ap V1h V10x) = \\ & (ap V9g (ap V0f V10x))))))))))))) \end{aligned}$$