

thm_2Ereal_topology_2ELIM_BILINEAR (TMQ2NDjWo3uuVcXgZS9oz7dMfZyo791nsQs)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal \tag{3}$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod ty_2Erealx_2Ereal ty_2Erealx_2Ereal)}) \tag{4}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 9 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ($

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \quad (7)$$

Definition 11 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (8)$$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)}) \quad (9)$$

Definition 12 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ ($

Definition 13 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet$

Definition 15 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Enet$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (13)$$

Definition 17 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^A$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (16)$$

Definition 18 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 19 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 20 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 21 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 24 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 25 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define $c_2Ereal_topology_2Elinear$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$

Definition 27 We define $c_2Ereal_topology_2Ebilinear$ to be $\lambda V0f \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0t1 \in A_27a. (\forall V1t2 \in A_27b. ((ap (\lambda V2x \in A_27b. \\ V0t1) V1t2) = V0t1))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ A_27a. (p \ V0t)) \Leftrightarrow (p \ V0t))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ (p \ V0t) \Rightarrow False) \Leftrightarrow \neg (p \ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg (\\ p \ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\ ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Erealx_2Ereal. (p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \\ V0x) \ V0x))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_topology_2EDist\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0x)\ V1y)) = (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub \\
& V0x)\ V1y))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\
& A_27a). ((p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually\ A_27a)\ (\\
& \lambda V1x \in A_27a.c_2Ebool_2ET))\ V0net)) \Leftrightarrow True))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0net \in (ty_2Ereal_topology_2Enet\ A.27a).(\forall V1P \in \\
& \quad (2^{A.27b}).(\forall V2h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}). \\
& \quad \quad (\forall V3f \in ((ty_2Erealax_2Ereal^{A.27a})^{A.27b}).(\forall V4g \in \\
& \quad \quad (ty_2Erealax_2Ereal^{A.27a})^{A.27b}).(\forall V5l \in (ty_2Erealax_2Ereal^{A.27b}). \\
& \quad \quad (\forall V6m \in (ty_2Erealax_2Ereal^{A.27b}).(\forall V7b1 \in ty_2Erealax_2Ereal. \\
& \quad \quad (\forall V8b2 \in ty_2Erealax_2Ereal.(((p\ (ap\ c_2Ereal_topology_2Ebilinear \\
& \quad \quad V2h)) \wedge ((p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually\ A.27a)\ (\lambda V9x \in \\
& \quad \quad A.27a.(ap\ (c_2Ebool_2E.21\ A.27b)\ (\lambda V10n \in A.27b.(ap\ (ap\ c_2Emin_2E.3D_3D_3E \\
& \quad \quad (ap\ V1P\ V10n))\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Eabs \\
& \quad \quad (ap\ V5l\ V10n)))\ V7b1))))))\ V0net)) \wedge ((p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually \\
& \quad \quad A.27a)\ (\lambda V11x \in A.27a.(ap\ (c_2Ebool_2E.21\ A.27b)\ (\lambda V12n \in \\
& \quad \quad A.27b.(ap\ (ap\ c_2Emin_2E.3D_3D_3E\ (ap\ V1P\ V12n))\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad \quad (ap\ c_2Ereal_2Eabs\ (ap\ V6m\ V12n)))\ V8b2))))))\ V0net)) \wedge ((\forall V13e \in \\
& \quad \quad ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))\ V13e)) \Rightarrow (p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually \\
& \quad \quad A.27a)\ (\lambda V14x \in A.27a.(ap\ (c_2Ebool_2E.21\ A.27b)\ (\lambda V15n \in \\
& \quad \quad A.27b.(ap\ (ap\ c_2Emin_2E.3D_3D_3E\ (ap\ V1P\ V15n))\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad \quad (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ (ap\ (ap\ V3f\ V15n) \\
& \quad \quad V14x))\ (ap\ V5l\ V15n))))\ V13e))))))\ V0net)))) \wedge (\forall V16e \in ty_2Erealax_2Ereal. \\
& \quad \quad ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))\ V16e)) \Rightarrow (p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually \\
& \quad \quad A.27a)\ (\lambda V17x \in A.27a.(ap\ (c_2Ebool_2E.21\ A.27b)\ (\lambda V18n \in \\
& \quad \quad A.27b.(ap\ (ap\ c_2Emin_2E.3D_3D_3E\ (ap\ V1P\ V18n))\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad \quad (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ (ap\ (ap\ V4g\ V18n) \\
& \quad \quad V17x))\ (ap\ V6m\ V18n))))\ V16e))))))\ V0net)))))) \Rightarrow (\forall V19e \in \\
& \quad \quad ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))\ V19e)) \Rightarrow (p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually \\
& \quad \quad A.27a)\ (\lambda V20x \in A.27a.(ap\ (c_2Ebool_2E.21\ A.27b)\ (\lambda V21n \in \\
& \quad \quad A.27b.(ap\ (ap\ c_2Emin_2E.3D_3D_3E\ (ap\ V1P\ V21n))\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\
& \quad \quad (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Ereal_2Ereal_sub\ (ap\ (ap\ V2h\ (ap \\
& \quad \quad (ap\ V3f\ V21n)\ V20x))\ (ap\ (ap\ V4g\ V21n)\ V20x)))\ (ap\ (ap\ V2h\ (ap\ V5l\ V21n)) \\
& \quad \quad (ap\ V6m\ V21n))))\ V19e))))))\ V0net))))))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\ & A_{27a}). (\forall V1h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}). \\ & (\forall V2f \in (ty_2Erealax_2Ereal^{A_{27a}}). (\forall V3g \in (ty_2Erealax_2Ereal^{A_{27a}}). \\ & (\forall V4l \in ty_2Erealax_2Ereal. (\forall V5m \in ty_2Erealax_2Ereal. \\ & (((p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E A_{27a}) V2f) V4l) \\ & V0net)) \wedge ((p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E A_{27a}) \\ & V3g) V5m) V0net)) \wedge (p (ap c_2Ereal_topology_2Ebilinear V1h)))))) \Rightarrow \\ & (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E A_{27a}) (\lambda V6x \in \\ & A_{27a}. (ap (ap V1h (ap V2f V6x)) (ap V3g V6x)))) (ap (ap V1h V4l) V5m)) \\ & V0net)))))))))) \end{aligned}$$