

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 13 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2. V0t))$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (12)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \quad (13)$$

Definition 17 We define $c_2Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Ereal_topology_2Eat) a)$.
 Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)} \quad (14)$$

Definition 18 We define $c_2Ereal_topology_2Ewithin$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Ewithin\ A_27a\ net)$.

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.(p\ V2t))))))$.

Definition 20 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Etrivial_limit\ A_27a\ net)$.

Definition 21 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Eeventually\ A_27a\ p\ net)$.

Definition 22 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A\ A_27a\ f)$.

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \vee (\neg(p\ V0t)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (26)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\
& 2.(((\forall V2x \in A_27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in \\
& A_27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \quad (31)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (36)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \forall A_{27c}. \text{nonempty } A_{27c} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1g \in (A_{27a}^{A_{27c}}). (\forall V2x \in A_{27c}. ((ap (ap (ap (c_{2Ecombin_2Eo} A_{27c} A_{27b} A_{27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in \text{ty_2Erealax_2Ereal}. ((ap \text{c_2Ereal_topology_2EDist} (ap (ap (c_{2Epair_2E_2C} \text{ty_2Erealax_2Ereal} \text{ty_2Erealax_2Ereal}) V0x) V0x)) = (ap \text{c_2Ereal_2Ereal_of_num} \text{c_2Enum_2E0}))) \quad (38)$$

Assume the following.

$$(\forall V0x \in \text{ty_2Erealax_2Ereal}. (\forall V1y \in \text{ty_2Erealax_2Ereal}. ((\neg(V0x = V1y)) \Leftrightarrow (p (ap (ap \text{c_2Erealax_2Ereal_lt} (ap \text{c_2Ereal_2Ereal_of_num} \text{c_2Enum_2E0})) (ap \text{c_2Ereal_topology_2EDist} (ap (ap (c_{2Epair_2E_2C} \text{ty_2Erealax_2Ereal} \text{ty_2Erealax_2Ereal}) V0x) V1y)))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1a \in ty_2Erealax_2Ereal. \\
& (\forall V2p \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Ereal_topology_2Eeventually \\
& ty_2Erealax_2Ereal) V2p) (ap (ap (c_2Ereal_topology_2Ewithin \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eat V1a)) V0s))) \Leftrightarrow \\
& (\exists V3d \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V3d)) \wedge (\forall V4x \in \\
& ty_2Erealax_2Ereal.(((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal \\
& V4x) V0s)) \wedge ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V4x) V1a)))) \wedge (p (ap (\\
& ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist (ap (\\
& ap (c_2Epair_2E2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V4x) \\
& V1a)))) \Rightarrow (p (ap V2p V4x)))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Eenet \\
& A_27a).(\forall V1p \in (2^{A_27a}).(\forall V2q \in (2^{A_27a}).((p (\\
& ap (ap (c_2Ereal_topology_2Eeventually A_27a) (\lambda V3x \in A_27a. \\
& (ap (ap c_2Ebool_2E2F_5C (ap V1p V3x)) (ap V2q V3x)))) V0net)) \Leftrightarrow \\
& ((p (ap (ap (c_2Ereal_topology_2Eeventually A_27a) V1p) V0net)) \wedge \\
& (p (ap (ap (c_2Ereal_topology_2Eeventually A_27a) V2q) V0net))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Eenet \\
& A_27a).(\forall V1p \in (2^{A_27a}).(\forall V2q \in (2^{A_27a}).(((\forall V3x \in \\
& A_27a.((p (ap V1p V3x)) \Rightarrow (p (ap V2q V3x)))) \wedge (p (ap (ap (c_2Ereal_topology_2Eeventually \\
& A_27a) V1p) V0net))) \Rightarrow (p (ap (ap (c_2Ereal_topology_2Eeventually \\
& A_27a) V2q) V0net))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (57)$$

Theorem 1

$$\begin{aligned} \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow & (\forall V0net \in (ty_2Ereal_topology_2Enet \\ & A_{27a}). (\forall V1f \in (ty_2Erealax_2Ereal^{A_{27a}}). (\forall V2g \in \\ & (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V3s \in (2^{ty_2Erealax_2Ereal}). \\ & (\forall V4y \in ty_2Erealax_2Ereal. (\forall V5z \in ty_2Erealax_2Ereal. \\ & (((p (ap (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E A_{27a}) V1f) V4y) \\ & V0net)) \wedge ((p (ap (ap (c_2Ereal_topology_2Eeventually A_{27a}) \\ & (\lambda V6w \in A_{27a}. (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN \\ & ty_2Erealax_2Ereal) (ap V1f V6w)) V3s)) (ap (ap c_2Emin_2E_3D_3D_3E \\ & (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) (ap V1f V6w)) V4y)) \\ & (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) (ap V2g V4y)) V5z)))))) \\ & V0net)) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) \\ & V2g) V5z) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) \\ & (ap c_2Ereal_topology_2Eat V4y)) V3s)))))) \Rightarrow (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\ & A_{27a}) (ap (ap (c_2Ecombin_2Eo A_{27a} ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V2g) V1f)) V5z) V0net))))))))) \end{aligned}$$