

thm_2Ereal__topology_2ELIM__CONG__AT (TMLyBps7JYjhQ47gS8agcCxbkQYW77XpEMm)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal \tag{3}$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod ty_2Erealx_2Ereal ty_2Erealx_2Ereal)}) \tag{4}$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$
Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{6}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{7}$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{8}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{9}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \tag{10}$$

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ($

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \tag{11}$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 14 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \tag{12}$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ereal_topology_2Emk_net\ A.27a \in ((ty_2Ereal_topology_2Enet\ A.27a)^{(2^{A-27a})^{A-27a}}) \tag{13}$$

Definition 15 We define $c_2Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Ereal_topology_2Eat) V0a)$.
Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet A_27a)} \quad (14)$$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) V0t1) V1t2))$

Definition 17 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Eat) A_27a$

Definition 18 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Eat) A_27a$

Definition 19 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A) A_27a$.
Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (22)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \quad (23)$$

$$(((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((\neg(V0x = V1y)) \Leftrightarrow (p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \ (ap \ c_2Ereal_topology_2EDist \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \ V0x) \ V1y))))))) \quad (24)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1l \in ty_2Erealax_2Ereal. (\forall V2a \in ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ (c_2Ereal_topology_2E_2D_2D_3E \ ty_2Erealax_2Ereal) \ V0f) \ V1l) \ (ap \ c_2Ereal_topology_2Eat \ V2a))) \Leftrightarrow (\forall V3e \in ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \ V3e)) \Rightarrow (\exists V4d \in ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \ V4d)) \wedge (\forall V5x \in ty_2Erealax_2Ereal. (((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \ (ap \ c_2Ereal_topology_2EDist \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \ V5x) \ V2a)))) \wedge (p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_topology_2EDist \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \ V5x) \ V2a))) \ V4d)))) \Rightarrow (p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_topology_2EDist \ (ap \ (ap \ (c_2Epair_2E_2C \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \ (ap \ V0f \ V5x)) \ V1l))) \ V3e)))))))))) \quad (25)$$

Theorem 1

$$(\forall V0a \in ty_2Erealax_2Ereal. (\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V3l \in ty_2Erealax_2Ereal. ((\forall V4x \in ty_2Erealax_2Ereal. ((\neg(V4x = V0a)) \Rightarrow ((ap \ V1f \ V4x) = (ap \ V2g \ V4x)))) \Rightarrow ((p \ (ap \ (ap \ (ap \ (c_2Ereal_topology_2E_2D_2D_3E \ ty_2Erealax_2Ereal) \ (\lambda V5x \in ty_2Erealax_2Ereal. (ap \ V1f \ V5x))) \ V3l) \ (ap \ c_2Ereal_topology_2Eat \ V0a))) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c_2Ereal_topology_2E_2D_2D_3E \ ty_2Erealax_2Ereal) \ V2g) \ V3l) \ (ap \ c_2Ereal_topology_2Eat \ V0a))))))))))$$