

thm_2Ereal_topology_2ELIM_DROP_LE
(TMHGzqjgLQBB3ojJRmv1hYf8EkEMDPb3G6m)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ **then** $(the (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal\ a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \tag{5}$$

Definition 7 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 10 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $ty_2Ereal_topology_2Eenet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ereal_topology_2Eenet A0) \quad (6)$$

Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in ((2^{A_27a})^{A_27a})(ty_2Ereal_topology_2Eenet A_27a) \quad (7)$$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 14 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Eenet$

Definition 15 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Eenet$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ereal_topology_2Eeventually$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (13)$$

Definition 18 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^A$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2E\ net \\ & \quad A_27a). (\forall V1f \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2g \in \\ & \quad (ty_2Erealax_2Ereal^{A_27a}). (\forall V3l \in ty_2Erealax_2Ereal. \\ & \quad (\forall V4m \in ty_2Erealax_2Ereal. (((\neg(p\ (ap\ (c_2Ereal_topology_2E\ trivial_limit \\ & \quad A_27a)\ V0net))) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E \\ & \quad A_27a)\ V1f)\ V3l)\ V0net)) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E \\ & \quad A_27a)\ V2g)\ V4m)\ V0net)) \wedge (p\ (ap\ (ap\ (c_2Ereal_topology_2E\ eventually \\ & \quad A_27a)\ (\lambda V5x \in A_27a. (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ V1f\ V5x)) \\ & \quad (ap\ V2g\ V5x)))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V3l \\ & \quad V4m)))))))))) \end{aligned} \quad (20)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0net \in (ty_2Ereal_topology_2Enet \\ & A_27a).(\forall V1f \in (ty_2Erealax_2Ereal^{A_27a}).(\forall V2g \in \\ & (ty_2Erealax_2Ereal^{A_27a}).(\forall V3l \in ty_2Erealax_2Ereal. \\ & (\forall V4m \in ty_2Erealax_2Ereal.(((\neg(p\ (ap\ (c_2Ereal_topology_2Etrivial_limit \\ & A_27a)\ V0net)))) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E \\ A_27a)\ V1f)\ V3l)\ V0net)) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E \\ A_27a)\ V2g)\ V4m)\ V0net)) \wedge (p\ (ap\ (ap\ (c_2Ereal_topology_2Eeventually \\ A_27a)\ (\lambda V5x \in A_27a.(ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ V1f\ V5x)) \\ (ap\ V2g\ V5x)))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V3l \\ V4m)))))))))) \end{aligned}$$