

thm_2Ereal__topology_2ELIM__IN__CLOSED__SET (TMaKU9qKWKu862nPfCMfkhpm8tZuS1ojHDt)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 4 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emin_2E_3D (2^{A_27a}))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (3)$$

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EDIFF\ s)\ t)$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 12 We define c_2Emin_2E40 to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E40\ a)\ ty_2Erealax_2Ereal)$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (ap\ (c_2Erealax_2Ereal_lt\ T1)\ T2)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (11)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (12)$$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ (c_2Ebool_2E_2$

Definition 18 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ c_2Ereal_topo$

Let $c_2Ereal_topology_2Eball : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eball \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (13)$$

Definition 19 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ ($

Let $ty_2Ereal_topology_2Eenet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Eenet\ A0) \quad (14)$$

Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Eenetord\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet\ A_27a)}) \quad (15)$$

Definition 20 We define $c_2Ebool_2E_5C_2E2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota. \lambda V0net \in (ty_2Ereal_topology$

Definition 22 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1net \in (ty_2$

Definition 23 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^A$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p\ V0t))))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2.(((\exists V2x \in A.27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg(p V0A)) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (p (ap (ap (c.2Ebool.2EIN A.27a) V0x) (c.2Epred_set.2EUNIV A.27a)))) \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\ (2^{A-27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EDIFF\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (\\ ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1y \in ty.2Erealx.2Ereal. \\ ((ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ ty.2Erealx.2Ereal)\ V0x)\ V1y)) = (ap\ c.2Ereal_topology.2EDist \\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ V1y)\ V0x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1y \in ty.2Erealx.2Ereal. \\ (\forall V2e \in ty.2Erealx.2Ereal. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal) \\ V1y)\ (ap\ c.2Ereal_topology.2Eball\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ ty.2Erealx.2Ereal)\ V0x)\ V2e)))) \Leftrightarrow (p\ (ap\ (ap\ c.2Erealx.2Ereal_lt \\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ ty.2Erealx.2Ereal)\ V0x)\ V1y)))\ V2e)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0s \in (2^{ty.2Erealx.2Ereal}). ((p\ (ap\ c.2Ereal_topology.2EOpen \\ V0s) \Leftrightarrow (\forall V1x \in ty.2Erealx.2Ereal. ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ ty.2Erealx.2Ereal)\ V1x)\ V0s) \Rightarrow (\exists V2e \in ty.2Erealx.2Ereal. \\ ((p\ (ap\ (ap\ c.2Erealx.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num \\ c.2Enum.2E0)\ V2e) \wedge (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ ty.2Erealx.2Ereal) \\ (ap\ c.2Ereal_topology.2Eball\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ ty.2Erealx.2Ereal)\ V1x)\ V2e)))\ V0s)))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0net \in (ty.2Ereal_topology.2Eenet \\ A.27a). (\forall V1p \in (2^{A-27a}). (\forall V2q \in (2^{A-27a}). ((p\ (\\ ap\ (ap\ (c.2Ereal_topology.2Eeventually\ A.27a)\ (\lambda V3x \in A.27a. \\ (ap\ (ap\ c.2Ebool.2E.2F.5C\ (ap\ V1p\ V3x))\ (ap\ V2q\ V3x))))\ V0net) \Leftrightarrow \\ ((p\ (ap\ (ap\ (c.2Ereal_topology.2Eeventually\ A.27a)\ V1p)\ V0net) \wedge \\ (p\ (ap\ (ap\ (c.2Ereal_topology.2Eeventually\ A.27a)\ V2q)\ V0net)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\ A_27a).(\forall V1p \in (2^{A_27a}).((\forall V2x \in A_27a.(\neg(p\ (ap \\ V1p\ V2x)))) \wedge (\neg(p\ (ap\ (c_2Ereal_topology_2Etrivial_limit\ A_27a) \\ V0net)))) \Rightarrow (\neg(p\ (ap\ (ap\ (c_2Ereal_topology_2Eventually\ A_27a) \\ V1p)\ V0net)))))) \end{aligned} \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (50)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee (\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p))) \Rightarrow (p V0p)) \quad (60)$$

Theorem 1

$$\begin{aligned} & \forall A.27a. \text{nonempty } A.27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\ & \quad A.27a). (\forall V1f \in (ty_2Erealax_2Ereal^{A.27a}). (\forall V2s \in \\ & \quad (2^{ty_2Erealax_2Ereal}). (\forall V3l \in ty_2Erealax_2Ereal. (\\ & ((p (ap c_2Ereal_topology_2EClosed V2s)) \wedge ((p (ap (ap (c_2Ereal_topology_2Eeventually \\ & \quad A.27a) (\lambda V4x \in A.27a. (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & \quad (ap V1f V4x)) V2s))) V0net)) \wedge (\neg(p (ap (c_2Ereal_topology_2Etrivial_limit \\ & \quad A.27a) V0net)))) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\ & \quad A.27a) V1f) V3l) V0net)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & \quad V3l) V2s)))))) \end{aligned}$$