

# thm\_2Ereal\_topology\_2ELIM\_SEQUENTIALLY (TMJktuwb3Tv1rTUtgiaS8QVqSaYNrVVzBYR)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap c\_2Enum\_2EREP\_num (ap c\_2Enum\_2ESUC\_REP m)))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 14** We define  $c\_Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 15** We define  $c\_Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Enet\ A0) \quad (5)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ A\_27a \in ((ty\_2Ereal\_topology\_2Enet\ A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (6)$$

**Definition 16** We define  $c\_2Ereal\_topology\_2Esequentially$  to be  $(ap\ (c\_2Ereal\_topology\_2Emk\_net\ ty\_2Ereal\_topology\_2Emk\_net$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (8)$$

**Definition 17** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epair\_2EABS\_prod$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (9)$$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (10)$$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_topology\_2Enet\ A\_27a)}) \quad (11)$$

**Definition 18** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota. \lambda V0net \in (ty\_2Ereal\_topology\_2Emk\_net$

**Definition 19** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{12}$$

**Definition 20** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ . Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{13}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{14}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \tag{15}$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ (ty\_2Erealax\_2Ereal\_REP\_CLASS\ a)))$ . Let  $c\_2Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \tag{16}$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 23** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^A)$ . Assume the following.

$$True \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \tag{21}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\exists V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \vee (\exists V4x \in A.27a. (p (ap V1Q V4x))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x)) \wedge (p V1Q)))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A.27a. (p\ (ap\ V1Q\ V3x))))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b)^{A.27a}. (\forall V4x \in A.27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in (2^{ty.2Enum.2Enum}). ((p\ (ap\ (ap\ (c.2Ereal\_topology.2Eventually\ ty.2Enum.2Enum)\ V0p)\ c.2Ereal\_topology.2Esequentially)) \Leftrightarrow (\exists V1N \in ty.2Enum.2Enum. (\forall V2n \in ty.2Enum.2Enum. (p\ (ap\ (ap\ c.2Earithmic.2E.3C.3D\ V1N)\ V2n)) \Rightarrow (p\ (ap\ V0p\ V2n)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (46)$$

### Theorem 1

$$\begin{aligned} & (\forall V0s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1l \in \\ & ty\_2Erealax\_2Ereal.((p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\ & ty\_2Enum\_2Enum) V0s) V1l) c\_2Ereal\_topology\_2Esequentially)) \Leftrightarrow \\ & (\forall V2e \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\ & (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V2e)) \Rightarrow (\exists V3N \in \\ & ty\_2Enum\_2Enum.(\forall V4n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\ & V3N) V4n)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist \\ & (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & (ap V0s V4n)) V1l))) V2e))))))))) \end{aligned}$$