

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge p\ x))))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge p\ x))))$

Definition 13 We define $c_2Ebool_2E_21$ to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge p\ x))))$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V3t3 \in A.\lambda V4t4 \in A.(ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge p\ x))))))$

Definition 17 We define c_2Ereal_2Emin to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge p\ x))))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (8)$$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge p\ x))))))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (13)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (14)$$

Definition 20 We define $c_2Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Ereal_topology_2Eat\ V0a))$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)} \quad (15)$$

Definition 21 We define $c_2Ereal_topology_2Ewithin$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 22 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 23 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 24 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^{A_27a})$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in \\ & A_27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p(ap V0P V2x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & (2^{A_27a}).((\forall V2x \in A_27a.((p(ap V0P V2x)) \wedge (p(ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A_27a.(p(ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p(\\ & ap V1Q V4x)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A_27a}).(((p V0P) \vee (\exists V2x \in A_27a.(p(ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ & A_27a.((p V0P) \vee (p(ap V1Q V3x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))))) \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (33)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (34)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27a}}).(\forall V2x \in A_{27a}.((p (ap (ap (c_{2Ebool_2EIN } A_{27a}) V2x) (ap (ap (c_{2Epred_set_2EUNION } A_{27a}) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_{2Ebool_2EIN } A_{27a}) V2x) V0s)) \vee (p (ap (ap (c_{2Ebool_2EIN } A_{27a}) V2x) V1t)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap (c_{2Erealax_2Ereal_lt } V2z) (ap (ap (c_{2Ereal_2Emin } V0x) V1y))) \Leftrightarrow ((p (ap (ap (c_{2Erealax_2Ereal_lt } V2z) V0x)) \wedge (p (ap (ap (c_{2Erealax_2Ereal_lt } V2z) V1y)))))))))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\
& \quad ty_2Erealax_2Ereal.(\forall V2a \in ty_2Erealax_2Ereal.(\forall V3s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).((p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\
& \quad ty_2Erealax_2Ereal) V0f) V1l) (ap (ap (c_2Ereal_topology_2Ewithin \\
& \quad ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eat V2a)) V3s))) \Leftrightarrow \\
& \quad (\forall V4e \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V4e)) \Rightarrow (\exists V5d \in \\
& \quad ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V5d)) \wedge (\forall V6x \in ty_2Erealax_2Ereal.(((p (\\
& \quad ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x) V3s)) \wedge ((p (ap (\\
& \quad ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& \quad (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) V6x) V2a)))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) V6x) V2a))) V5d)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) (ap V0f V6x)) V1l))) V4e))))))))))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& \quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& \quad p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& \quad ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
& \hspace{15em} (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{47}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1x \in \\
& ty_2Erealax_2Ereal. (\forall V2l \in ty_2Erealax_2Ereal. (\forall V3s \in \\
& (2^{ty_2Erealax_2Ereal}). (\forall V4t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p \ (ap \ (ap \ (ap \ (ap \ (c_2Ereal_topology_2E_2D_2D_3E \ ty_2Erealax_2Ereal) \\
& V0f) \ V2l) \ (ap \ (ap \ (c_2Ereal_topology_2Ewithin \ ty_2Erealax_2Ereal) \\
& (ap \ c_2Ereal_topology_2Eat \ V1x)) \ V3s))) \wedge (p \ (ap \ (ap \ (ap \ (c_2Ereal_topology_2E_2D_2D_3E \\
& ty_2Erealax_2Ereal) \ V0f) \ V2l) \ (ap \ (ap \ (c_2Ereal_topology_2Ewithin \\
& ty_2Erealax_2Ereal) \ (ap \ c_2Ereal_topology_2Eat \ V1x)) \ V4t)))))) \Rightarrow \\
& (p \ (ap \ (ap \ (ap \ (c_2Ereal_topology_2E_2D_2D_3E \ ty_2Erealax_2Ereal) \\
& V0f) \ V2l) \ (ap \ (ap \ (c_2Ereal_topology_2Ewithin \ ty_2Erealax_2Ereal) \\
& (ap \ c_2Ereal_topology_2Eat \ V1x)) \ (ap \ (ap \ (c_2Epred_set_2EUNION \\
& ty_2Erealax_2Ereal) \ V3s) \ V4t))))))))))
\end{aligned}$$