

thm_2Ereal__topology_2ELIM__WITHIN__OPEN
(TM-
NYM3VWsymm92R6T5LqoopbSahPGUkwsTH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 8 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** *(the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).*

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \quad (7)$$

Definition 10 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 12 We define c_2Ebool_2E3F to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40$

Definition 13 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E2$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (12)$$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in ((2^{A_27a})^{A_27a})(ty_2Ereal_topology_2Enet\ A_27a) \quad (13)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \quad (14)$$

Definition 14 We define $c_2Ereal_topology_2Ewithin$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet$

Definition 15 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 17 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 18 We define $c_2Ereal_topology_2Eat$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Ereal_topology_2E$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t$

Definition 20 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet$

Definition 21 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal$

Definition 22 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in \\ & A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in \\ & A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ & A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.(p (ap V0P V2x)) \wedge (p V1Q)) \Leftrightarrow ((\exists V3x \in \\ & A.27a.(p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A.27a.(p\ (ap\ V1Q\ V3x))))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in A.27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).(\forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal.(\forall V1y \in ty_2Erealx_2Ereal.(\forall V2z \in ty_2Erealx_2Ereal.(((p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ V0x)\ V2z)))))) \quad (38)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal.(\forall V1y \in ty_2Erealx_2Ereal.(((p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ (ap\ c.2Ereal_2Ereal_of_num\ c.2Enum_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ (ap\ c.2Ereal_2Ereal_of_num\ c.2Enum_2E0))\ V1y))) \Rightarrow (\exists V2z \in ty_2Erealx_2Ereal.((p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ (ap\ c.2Ereal_2Ereal_of_num\ c.2Enum_2E0))\ V2z)) \wedge ((p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ V2z)\ V0x)) \wedge (p\ (ap\ (ap\ c.2Erealx_2Ereal_lt\ V2z)\ V1y)))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\
& \quad ty_2Erealax_2Ereal.(\forall V2a \in ty_2Erealax_2Ereal.(\forall V3s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).((p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\
& \quad ty_2Erealax_2Ereal) V0f) V1l) (ap (ap (c_2Ereal_topology_2Ewithin \\
& \quad ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eat V2a)) V3s))) \Leftrightarrow \\
& \quad (\forall V4e \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V4e)) \Rightarrow (\exists V5d \in \\
& \quad ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V5d)) \wedge (\forall V6x \in ty_2Erealax_2Ereal.(((p (\\
& \quad ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x) V3s)) \wedge ((p (ap (\\
& \quad ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& \quad (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) V6x) V2a)))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) V6x) V2a))) V5d)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) (ap V0f V6x)) V1l))) V4e))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\
& \quad ty_2Erealax_2Ereal.(\forall V2a \in ty_2Erealax_2Ereal.((p (ap \\
& \quad (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) \\
& \quad V0f) V1l) (ap c_2Ereal_topology_2Eat V2a))) \Leftrightarrow (\forall V3e \in ty_2Erealax_2Ereal. \\
& \quad ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V3e)) \Rightarrow (\exists V4d \in ty_2Erealax_2Ereal.((p (ap \\
& \quad (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& \quad V4d)) \wedge (\forall V5x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Ereal_topology_2EDist \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V5x) V2a)))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V5x) V2a))) V4d)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad (ap V0f V5x)) V1l))) V3e))))))))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\
& \quad ty_2Erealax_2Ereal.(\forall V2a \in ty_2Erealax_2Ereal.(\forall V3s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).((p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\
& \quad ty_2Erealax_2Ereal) V0f) V1l) (ap c_2Ereal_topology_2Eat V2a))) \Rightarrow \\
& \quad (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) \\
& \quad V0f) V1l) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) \\
& \quad (ap c_2Ereal_topology_2Eat V2a)) V3s)))))) \\
& \hspace{15em} (42)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\ & ty_2Erealax_2Ereal.(\forall V2a \in ty_2Erealax_2Ereal.(\forall V3s \in \\ & (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & V2a) V3s)) \wedge (p (ap c_2Ereal_topology_2EOpen V3s))) \Rightarrow ((p (ap (ap \\ & (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Erealax_2Ereal) V0f) \\ & V1l) (ap (ap (c_2Ereal_topology_2Ewithin ty_2Erealax_2Ereal) \\ & (ap c_2Ereal_topology_2Eat V2a)) V3s))) \Leftrightarrow (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\ & ty_2Erealax_2Ereal) V0f) V1l) (ap c_2Ereal_topology_2Eat V2a))))))))) \end{aligned}$$