

thm_2Ereal_topology_2ELINEAR__BOUNDED (TMR6gvFY6wfUcXZyK7S17XimvBBLEUzcKJa)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ P))$

Definition 4 We define `c_2Ebool_2E_T` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{1}$$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})) \ P))$

Definition 6 We define `c_2Ebool_2E_F` to be $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t))$.

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_F))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2.V2t))$

Let `c_2Enum_2EREP_num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\text{omega}^{ty_2Enum_2Enum}) \tag{2}$$

Let `c_2Enum_2ESUC_REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\text{omega}^{\text{omega}}) \tag{3}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \tag{4}$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (6)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (7)$$

Definition 16 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 19 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (12)$$

Definition 21 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (15)$$

Definition 22 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 23 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum} \quad (16)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 24 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 25 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 27 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (30)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c.nonempty A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}).(\forall V2x \in A_27c.((ap (ap (ap (c.2Ecombin_2Eo A_27c A_27b A_27a) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (32)$$

Assume the following.

$$p \left(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p \left(\text{ap} \left(\text{c_2Epred_set_2EFINITE } ty_2Enum_2Enum \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E V0m \right) V1n \right) \right) \right) \right) \quad (33)$$

Assume the following.

$$\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p \left(\text{ap} \left(\text{ap} \left(\text{c_2Ebool_2EIN } ty_2Enum_2Enum \right) V2p \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E V0m \right) V1n \right) \right) \Leftrightarrow ((p \left(\text{ap} \left(\text{ap } c_2Earithmetic_2E_3C_3D V0m \right) V2p \right) \wedge (p \left(\text{ap} \left(\text{ap } c_2Earithmetic_2E_3C_3D V2p \right) V1n \right) \right) \right) \right) \right) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V1c \in ty_2Erealax_2Ereal. (\forall V2s \in (2^{A_27a}). ((\text{ap} \left(\text{ap} \left(\text{c_2Eiterate_2ESum } A_27a \right) V2s \right) (\lambda V3x \in A_27a. (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul V1c \right) \left(\text{ap } V0f V3x \right) \right) \right) = (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul V1c \right) \left(\text{ap} \left(\text{ap} \left(\text{c_2Eiterate_2ESum } A_27a \right) V2s \right) V0f \right) \right) \right) \right) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). (((p \left(\text{ap} \left(\text{c_2Epred_set_2EFINITE } A_27a \right) V2s \right) \wedge (\forall V3x \in A_27a. ((p \left(\text{ap} \left(\text{ap} \left(\text{c_2Ebool_2EIN } A_27a \right) V3x \right) V2s \right) \Rightarrow (p \left(\text{ap} \left(\text{ap } c_2Ereal_2Ereal_lte \left(\text{ap } c_2Ereal_2Eabs \left(\text{ap } V0f V3x \right) \right) \left(\text{ap } V1g V3x \right) \right) \right) \right) \Rightarrow (p \left(\text{ap} \left(\text{ap } c_2Ereal_2Ereal_lte \left(\text{ap } c_2Ereal_2Eabs \left(\text{ap} \left(\text{ap} \left(\text{c_2Eiterate_2ESum } A_27a \right) V2s \right) V0f \right) \right) \left(\text{ap} \left(\text{ap} \left(\text{c_2Eiterate_2ESum } A_27a \right) V2s \right) V1g \right) \right) \right) \right) \right) \right) \quad (36)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1n \in ty_2Enum_2Enum. ((\text{ap} \left(\text{ap} \left(\text{c_2Eiterate_2ESum } ty_2Enum_2Enum \right) V1n \right) \left(\text{ap} \left(\text{ap } c_2Eiterate_2E_2E V1n \right) V0f \right) = (\text{ap } V0f V1n))) \quad (37)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((\text{ap} \left(\text{ap } c_2Erealax_2Ereal_add \left(\text{ap } c_2Erealax_2Ereal_neg V0x \right) V0x \right) = (\text{ap } c_2Ereal_2Ereal_of_num \ c_2Enum_2E0))) \quad (38)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul V0x \right) V1y) = (\text{ap} \left(\text{ap } c_2Erealax_2Ereal_mul V1y \right) V0x))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V0x) V0x)))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \Leftrightarrow ((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad V0x) V1y)) \vee (V0x = V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V1y) V0x))) \Leftrightarrow (V0x = V1y))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y)) = \\
& (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Eabs\ V0x))\ (ap\ c_2Ereal_2Eabs \\
& \quad V1y))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& V1y) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad V0x)\ V1y))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V1x)\ V0y)) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad V0y)\ V1x))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ c_2Erealax_2Ereal_neg\ V0x)) \\
& (ap\ c_2Erealax_2Ereal_neg\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad V1y)\ V0x))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ c_2Erealax_2Ereal_neg \\
& \quad (ap\ c_2Erealax_2Ereal_neg\ V0x)) = V0x))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ (ap\ c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ c_2Erealax_2Ereal_add \\
& \quad V0x)\ V1y))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1c \in \\
& ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. ((p (ap \\
& c_2Ereal_topology_2Elinear V0f)) \Rightarrow ((ap V0f (ap (ap c_2Erealax_2Ereal_mul \\
& V1c) V2x)) = (ap (ap c_2Erealax_2Ereal_mul V1c) (ap V0f V2x))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \\
& (\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). \\
& (((p (ap c_2Ereal_topology_2Elinear V0f)) \wedge (p (ap (c_2Epred_set_2EFINITE \\
& A_27a) V2s))) \Rightarrow ((ap V0f (ap (ap (c_2Eiterate_2ESum A_27a) V2s) V1g)) = \\
& (ap (ap (c_2Eiterate_2ESum A_27a) V2s) (ap (ap (c_2Ecombin_2Eo \\
& A_27a) ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1g))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{61}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{72}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). ((p \ (\\
& ap \ c_2Ereal_topology_2Elinear \ V0f)) \Rightarrow (\exists V1B \in ty_2Erealax_2Ereal. \\
& (\forall V2x \in ty_2Erealax_2Ereal. (p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \\
& (ap \ c_2Ereal_2Eabs \ (ap \ V0f \ V2x)) \ (ap \ (ap \ c_2Erealax_2Ereal_mul \\
& V1B) \ (ap \ c_2Ereal_2Eabs \ V2x)))))))))
\end{aligned}$$