

# thm\_2Ereal\_\_topology\_2ELINEAR\_\_INDEPENDENT\_\_EXTEND (TMTOhCE3gBVzPETZStcVo4ftZ8653fdGqkd)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 4** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{ty\_2Erealax}) \tag{4}$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 8** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})))$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap (c\_2Emin\_2E\_40 (ty\_2Erealax\_2Ereal\_REP$

Let  $c\_2Erealax\_2Ereal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) \quad (8)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 13** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (c\_2Ebool\_2E\_2F\_5C t1 t2))))$

**Definition 15** We define  $c\_2Ereal\_topology\_2Elinear$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 17** We define  $c\_2\text{Earithmetic\_2EZERO}$  to be  $c\_2\text{Enum\_2E0}$ .

Let  $c\_2\text{Enum\_2EREP\_num} : \iota$  be given. Assume the following.

$$c\_2\text{Enum\_2EREP\_num} \in (\text{omega}^{ty\_2\text{Enum\_2Enum}}) \quad (12)$$

Let  $c\_2\text{Enum\_2ESUC\_REP} : \iota$  be given. Assume the following.

$$c\_2\text{Enum\_2ESUC\_REP} \in (\text{omega}^{\text{omega}}) \quad (13)$$

**Definition 18** We define  $c\_2\text{Enum\_2ESUC}$  to be  $\lambda V0m \in ty\_2\text{Enum\_2Enum} . (ap\ c\_2\text{Enum\_2EABS\_num})$

Let  $c\_2\text{Earithmetic\_2E\_2B} : \iota$  be given. Assume the following.

$$c\_2\text{Earithmetic\_2E\_2B} \in ((ty\_2\text{Enum\_2Enum}^{ty\_2\text{Enum\_2Enum}})^{ty\_2\text{Enum\_2Enum}}) \quad (14)$$

**Definition 19** We define  $c\_2\text{Earithmetic\_2EBIT1}$  to be  $\lambda V0n \in ty\_2\text{Enum\_2Enum} . (ap\ (ap\ c\_2\text{Earithmetic\_2E\_2B}))$

**Definition 20** We define  $c\_2\text{Earithmetic\_2ENUMERAL}$  to be  $\lambda V0x \in ty\_2\text{Enum\_2Enum} . V0x$ .

Let  $c\_2\text{Epred\_set\_2ECARD} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a . \text{nonempty } A\_27a \Rightarrow c\_2\text{Epred\_set\_2ECARD } A\_27a \in (ty\_2\text{Enum\_2Enum}^{(2^{A\_27a})}) \quad (15)$$

**Definition 21** We define  $c\_2\text{Ebool\_2EF}$  to be  $(ap\ (c\_2\text{Ebool\_2E\_21 } 2)) (\lambda V0t \in 2 . V0t)$ .

**Definition 22** We define  $c\_2\text{Ebool\_2E\_7E}$  to be  $(\lambda V0t \in 2 . (ap\ (ap\ c\_2\text{Emin\_2E\_3D\_3D\_3E } V0t))\ c\_2\text{Ebool\_2E\_21 } 2)$

**Definition 23** We define  $c\_2\text{Ebool\_2E\_3F}$  to be  $\lambda A\_27a : \iota . (\lambda V0P \in (2^{A\_27a}) . (ap\ V0P\ (ap\ (c\_2\text{Emin\_2E\_40 } V0P))))$

**Definition 24** We define  $c\_2\text{Eprim\_rec\_2E\_3C}$  to be  $\lambda V0m \in ty\_2\text{Enum\_2Enum} . \lambda V1n \in ty\_2\text{Enum\_2Enum} . V1n$

**Definition 25** We define  $c\_2\text{Ebool\_2E\_5C\_2F}$  to be  $(\lambda V0t1 \in 2 . (\lambda V1t2 \in 2 . (ap\ (c\_2\text{Ebool\_2E\_21 } 2)) (\lambda V2t \in 2 . V2t))))$

**Definition 26** We define  $c\_2\text{Earithmetic\_2E\_3C\_3D}$  to be  $\lambda V0m \in ty\_2\text{Enum\_2Enum} . \lambda V1n \in ty\_2\text{Enum\_2Enum} . V1n$

**Definition 27** We define  $c\_2\text{Ebool\_2EIN}$  to be  $\lambda A\_27a : \iota . (\lambda V0x \in A\_27a . (\lambda V1f \in (2^{A\_27a}) . (ap\ V1f\ V0x)))$

**Definition 28** We define  $c\_2\text{Epred\_set\_2ESUBSET}$  to be  $\lambda A\_27a : \iota . \lambda V0s \in (2^{A\_27a}) . \lambda V1t \in (2^{A\_27a}) . (ap\ (c\_2\text{Ebool\_2E\_3F } V1t))\ V0s$

Let  $c\_2\text{Ereal\_2Ereal\_of\_num} : \iota$  be given. Assume the following.

$$c\_2\text{Ereal\_2Ereal\_of\_num} \in (ty\_2\text{Erealax\_2Ereal}^{ty\_2\text{Enum\_2Enum}}) \quad (16)$$

**Definition 29** We define  $c\_2\text{Ereal\_topology\_2Esubspace}$  to be  $\lambda V0s \in (2^{ty\_2\text{Erealax\_2Ereal}}) . (ap\ (ap\ c\_2\text{Ebool\_2E\_3F } V0s))$

Let  $c\_2\text{Epair\_2EABS\_prod} : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a . \text{nonempty } A\_27a \Rightarrow \forall A\_27b . \text{nonempty } A\_27b \Rightarrow c\_2\text{Epair\_2EABS\_prod } A\_27a\ A\_27b \in ((ty\_2\text{Epair\_2Eprod } A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (17)$$

**Definition 30** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(18)

**Definition 31** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 32** We define  $c\_2Etopology\_2Ehull$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 33** We define  $c\_2Ereal\_topology\_2Espan$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (ap (c\_2Etopology\_2Ehull : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 34** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 35** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 36** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 37** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 38** We define  $c\_2Ereal\_topology\_2Edependent$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2EF : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 39** We define  $c\_2Ereal\_topology\_2Eindependent$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2EF : \iota \Rightarrow \iota \Rightarrow \iota)$

**Definition 40** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 : \iota \Rightarrow \iota \Rightarrow \iota)$

Assume the following.

$$True$$
(19)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))$$
(20)

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t)))$$
(21)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))$$
(22)

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t))))$$
(23)

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p(ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p(ap V1Q V3x)))))) \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\ & ((\exists V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \vee (\exists V4x \in A.27a.(p \ ( \\ & \quad \quad \quad ap \ V1Q \ V4x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q)) \Leftrightarrow (\exists V3x \in \\ & \quad \quad \quad A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).((p \ V0P) \vee (\exists V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in \\ & \quad \quad \quad A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\ & \quad \quad \quad A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ & \quad \quad \quad V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A.27a}).((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ & \quad \quad \quad V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ & (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ & \quad \quad \quad (p \ V0A)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\ & p \ V0A) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge (\neg(p \ V1B)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (43)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow 2.(((p V0x) \Leftrightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b)^{A\_27a}).(\forall V4x \in A\_27a.(p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (46)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t))))))) \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) (c\_2Epred\_set\_2EUNIV A\_27a)))) \quad (48)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) V0s) (c\_2Epred\_set\_2EUNIV A\_27a)))) \quad (49)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) (c\_2Epred\_set\_2EUNIV A\_27a)) V0s)) \Leftrightarrow (V0s = (c\_2Epred\_set\_2EUNIV A\_27a)))) \quad (50)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2Eindependent V0s)) \Rightarrow ((p (ap (c\_2Epred\_set\_2EFINITE ty\_2Erealax\_2Ereal) V0s)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (c\_2Epred\_set\_2ECARD ty\_2Erealax\_2Ereal) V0s)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1v \in (2^{ty\_2Erealax\_2Ereal}). \\
& (((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V0s) \\
& V1v)) \wedge (p (ap c\_2Ereal\_topology\_2Eindependent V0s))) \Rightarrow (\exists V2b \in \\
& (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& ty\_2Erealax\_2Ereal) V0s) V2b)) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& ty\_2Erealax\_2Ereal) V2b) V1v)) \wedge ((p (ap c\_2Ereal\_topology\_2Eindependent \\
& V2b)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& V1v) (ap c\_2Ereal\_topology\_2Espan V2b))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1b \in \\
& (2^{ty\_2Erealax\_2Ereal}).((p (ap (c\_2Epred\_set\_2EFINITE ty\_2Erealax\_2Ereal) \\
& V1b)) \Rightarrow ((p (ap c\_2Ereal\_topology\_2Eindependent V1b)) \Rightarrow (\exists V2g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((\forall V3x \in ty\_2Erealax\_2Ereal. \\
& (\forall V4y \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V3x) (ap c\_2Ereal\_topology\_2Espan V1b))) \wedge \\
& (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4y) (ap c\_2Ereal\_topology\_2Espan \\
& V1b)))) \Rightarrow ((ap V2g (ap (ap c\_2Erealax\_2Ereal\_add V3x) V4y)) = (ap \\
& (ap c\_2Erealax\_2Ereal\_add (ap V2g V3x) (ap V2g V4y)))))) \wedge ((\forall V5x \in \\
& ty\_2Erealax\_2Ereal.(\forall V6c \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5x) (ap c\_2Ereal\_topology\_2Espan \\
& V1b))) \Rightarrow ((ap V2g (ap (ap c\_2Erealax\_2Ereal\_mul V6c) V5x)) = (ap \\
& (ap c\_2Erealax\_2Ereal\_mul V6c) (ap V2g V5x)))))) \wedge ((\forall V7x \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V7x) V1b)) \Rightarrow ((ap V2g V7x) = (ap V0f V7x))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{58}$$



Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{63}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1b \in \\
& (2^{ty\_2Erealax\_2Ereal}). ((p (ap c\_2Ereal\_topology\_2EIndependent \\
& V1b)) \Rightarrow (\exists V2g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap c\_2Ereal\_topology\_2Elinear V2g)) \wedge (\forall V3x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V3x) V1b)) \Rightarrow ((ap \\
& V2g V3x) = (ap V0f V3x))))))
\end{aligned}$$