thm_2Ereal__topology_2ELOCALLY__COMPACT__COMPACT (TMRt2HpsDmbLbQNG5gkcrfva3n5LRvnuY6j)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A\wedge p$ *of type* $\iota\Rightarrow\iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota\Rightarrow\iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Epair\_2Eprod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P\Rightarrow p\ Q)$ *of type* ι.

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 7 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 14 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{5}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{7}$$

Definition 15 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{8}$$

Definition 16 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 18 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 19 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E_2

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{13}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered}\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in \\ ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})})\end{gathered} \tag{14}$$

Definition 20 *We define* c_2Ereal__topology_2Eeuclidean *to be* $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Let $c\_2Etopology\_2Eopen\_\_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{gathered}\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_\_in\ A\_27a \in \\ ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology\ A\_27a)})\end{gathered} \tag{15}$$

Definition 21 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 22 *We define* c_2Ereal__topology_2Esubtopology *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Eto$

Definition 23 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 24 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{16}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{17}$$

Definition 25 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 26 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 27 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 28 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Ereal\_\_topology\_2Enet : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Ereal\_\_topology\_2Enet\ A0) \tag{18}$$

Let $c\_2Ereal\_\_topology\_2Emk\_\_net : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a \in ((ty\_2Ereal\_\_topology\_2Enet\ A\_27a)^{((2^{A\_27a})^{A\_27a})}) \tag{19}$$

Definition 29 *We define* c_2Ereal__topology_2Esequentially *to be* $(ap\ (c\_2Ereal\_\_topology\_2Emk\_\_net\ ty\_2$

Definition 30 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Ereal\_\_topology\_2Enetord : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ereal\_\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_\_topology\_2Enet\ A\_27a)}) \tag{20}$$

Definition 31 *We define* c_2Ereal__topology_2Etrivial__limit *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topolog$

Definition 32 *We define* c_2Ereal__topology_2Eeventually *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2$

Definition 33 *We define* c_2Ereal__topology_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A}$

Definition 34 *We define* c_2Ereal__topology_2Ecompact *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2

Definition 35 *We define* c_2Ereal__topology_2Elocally *to be* $\lambda V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})}).\lambda V1s \in (2^{ty\_2Ere}$

Definition 36 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x)))))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \Rightarrow (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p\ V0P) \Rightarrow (p\ (ap\ V1Q\ V3x))))))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s))))) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1f \in (A\_27a^{A\_27b}).(\forall V2s \in (2^{A\_27b}).((\forall V3y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27b\ A\_27a)\ V1f)\ V2s))) \Rightarrow (p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x))))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1f)\ V0s)))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1sos \in (2^{(2^{A\_27a})}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2s)\ V1sos))))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}).(\forall V1f \in (A\_27a^{A\_27b}).(\forall V2s \in (2^{A\_27b}).((\exists V3t \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V3t)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V3t)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27b\ A\_27a)\ V1f)\ V2s))) \wedge (p\ (ap\ V0P\ V3t))))) \Leftrightarrow (\exists V4t \in (2^{A\_27b}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ V4t)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27b)\ V4t)\ V2s)) \wedge (p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27b\ A\_27a)\ V1f)\ V4t)))))))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((2^{A\_27b})^{A\_27a}).(\forall V1s \in (2^{A\_27a}).((ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ (2^{A\_27b}))\ V0f)\ V1s)) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ A\_27b)\ (\lambda V2y \in A\_27b.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ V2y)\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ A\_27a)\ (\lambda V3x \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V2y)\ (ap\ V0f\ V3x)))))))))))) \tag{51}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ V0s))) \tag{52}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1u \in (2^{ty\_2Erealax\_2Ereal}). (\forall V2v \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1u))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ V1u)\ V0s))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V2v)\ V1u))) \Rightarrow (p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2v))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ V2v)\ V0s))))))) \tag{53}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ V0s))\Leftrightarrow(\forall V1f \in (2^{(2^{ty\_2Erealax\_2Ereal})}).(((\forall V2t \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Erealax\_2Ereal}))\ V2t)\ V1f))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ V2t))))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V0s)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ ty\_2Erealax\_2Ereal)\ V1f))))\Rightarrow(\exists V3f\_27 \in (2^{(2^{ty\_2Erealax\_2Ereal})}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (2^{ty\_2Erealax\_2Ereal}))\ V3f\_27)\ V1f))\wedge((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ (2^{ty\_2Erealax\_2Ereal}))\ V3f\_27))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V0s)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ ty\_2Erealax\_2Ereal)\ V3f\_27)))))))))) \tag{54}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V0a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))))) \tag{55}$$

Assume the following.

$$(\forall V0s \in (2^{(2^{ty\_2Erealax\_2Ereal})}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ (2^{ty\_2Erealax\_2Ereal}))\ V0s))\wedge(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{ty\_2Erealax\_2Ereal}))\ V1t)\ V0s))\Rightarrow(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ V1t)))))\Rightarrow(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ ty\_2Erealax\_2Ereal)\ V0s))))) \tag{56}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Elocally}\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact})\ V0s))\Leftrightarrow(\forall V1x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V1x)\ V0s))\Rightarrow(\exists V2u \in (2^{ty\_2Erealax\_2Ereal}).(\exists V3v \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V1x)\ V2u))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V2u)\ V3v))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V3v)\ V0s))\wedge((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ V2u))\wedge(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ V3v)))))))))))) \tag{57}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ V1s))\wedge(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ V2t)))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V1s)\ V2t))))))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1k \in (2^{(2^{A\_27a})}).((\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2s)\ V1k))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ V2s))))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ V1k)))))) \tag{74}$$

Theorem 1

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Elocally}\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact})\ V0s))\Leftrightarrow(\forall V1k \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V1k)\ V0s))\wedge(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ V1k)))\Rightarrow(\exists V2u \in (2^{ty\_2Erealax\_2Ereal}).(\exists V3v \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V1k)\ V2u))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V2u)\ V3v))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V3v)\ V0s))\wedge((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ V2u))\wedge(p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ecompact}\ V3v)))))))))))))$$