

thm_2Ereal__topology_2ELOCALLY__COMPACT__COMPACT__SU
 (TMbb7gu52YDeEr1kQ6Ce41P4cCGr14XeaCe)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2T)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emin_2E_3D (2^{A_27a}))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 9 We define $c_2Epred_set_2EINTER$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Ereal_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (the\ (\lambda x.x \in A \wedge p\ x))\ of\ type\ \iota \Rightarrow \iota))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 14 We define c_2Ebool_2E3F to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ (the\ (\lambda x.x \in A \wedge p\ x))\ of\ type\ \iota \Rightarrow \iota))$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E3F\ (the\ (\lambda x.x \in A \wedge p\ x))\ of\ type\ \iota \Rightarrow \iota))$

Definition 16 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E))$

Definition 18 We define $c_Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E))$

Definition 19 We define $c_Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo)$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (13)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (15)$$

Definition 20 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 21 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal)$

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 23 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))))$

Definition 24 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND) (ap (c_2Erealax_2Ereal)) (ap (c_2Erealax_2Ereal))))))$

Definition 25 We define $c_2Ereal_topology_2Ebounded_def$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2ECOND) (ap (c_2Erealax_2Ereal)))$

Definition 26 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in (2^{ty_2Erealax_2Ereal})$

Definition 27 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 28 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EUNION))$

Definition 29 We define $c_2Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap (c_2Epred_set_2EUNION) (ap (c_2Erealax_2Ereal))))$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (16)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in \\ ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})}})) \end{aligned} \quad (17)$$

Definition 30 We define $c_Ereal_topology_2Euclidean$ to be (ap (c_Etopology_2Etopology ty_2Erealax

Let $c_Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Etopology_2Eopen_in\ A_27a \in ((2^{(2^{A-27a})})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (18)$$

Definition 31 We define $c_Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 32 We define $c_EPred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$

Let $c_Eenum_2EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_2EREP_num \in (\omega^{ty_2Eenum_2Eenum}) \quad (19)$$

Let $c_Eenum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_2ESUC_REP \in (\omega^{\omega}) \quad (20)$$

Definition 33 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum.(ap\ c_Eenum_2EABS_num$

Definition 34 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 35 We define $c_Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 36 We define $c_Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (21)$$

Let $c_Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ereal_topology_2Emk_net\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A-27a})^{A-27a}}) \quad (22)$$

Definition 37 We define $c_Ereal_topology_2Esequentially$ to be (ap (c_Ereal_topology_2Emk_net ty_2E

Definition 38 We define $c_Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A-27c}).\lambda V1$

Let $c_Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ereal_topology_2Enetord\ A_27a \in (((2^{A-27a})^{A-27a})^{(ty_2Ereal_topology_2Enet\ A_27a)}) \quad (23)$$

Definition 39 We define $c_Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology$

Definition 40 We define $c_Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A-27a}).\lambda V1net \in (ty_2E$

Definition 41 We define $c_Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 42 We define $c_Ereal_topology_Ecompact$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_Ebool_2E$

Definition 43 We define $c_Ereal_topology_Elocally$ to be $\lambda V0P \in (2^{(2^{ty_2Erealax_2Ereal})}).\lambda V1s \in (2^{ty_2Ere$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (38)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1Q \in 2.(((\exists V2x \in A_{.27a}.(p (ap V0P V2x)) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in A_{.27a}.((p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (39)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(\forall V2u \in (2^{A_{.27a}}).(((p (ap (ap (c_{.2Epred_set_2ESUBSET} A_{.27a}) V0s) V1t)) \wedge (p (ap (ap (c_{.2Epred_set_2ESUBSET} A_{.27a}) V1t) V2u))) \Rightarrow (p (ap (ap (c_{.2Epred_set_2ESUBSET} A_{.27a}) V0s) V2u)))))) \quad (40)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p (ap (ap (c_{.2Epred_set_2ESUBSET} A_{.27a}) V0s) (c_{.2Epred_set_2EUNIV} A_{.27a})))) \quad (41)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) V2x) (ap (ap (c_{.2Epred_set_2EINTER} A_{.27a}) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) V2x) V0s)) \wedge (p (ap (ap (c_{.2Ebool_2EIN} A_{.27a}) V2x) V1t)))))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\forall V0s \in (2^{A_{27a}}). (\forall V1t \in \\
& (2^{A_{27a}}). (p (ap (ap (ap (c_{2E}pred_set_{2E}SUBSET A_{27a}) (ap (ap (c_{2E}pred_set_{2E}INTER \\
& A_{27a}) V0s) V1t)) V0s)))) \wedge (\forall V2s \in (2^{A_{27a}}). (\forall V3t \in \\
& (2^{A_{27a}}). (p (ap (ap (ap (c_{2E}pred_set_{2E}SUBSET A_{27a}) (ap (ap (c_{2E}pred_set_{2E}INTER \\
& A_{27a}) V3t) V2s)) V2s))))))
\end{aligned} \tag{43}$$

Assume the following.

$$(p (ap c_{2E}real_topology_{2E}Open (c_{2E}pred_set_{2E}UNIV ty_{2E}realax_{2E}real))) \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_{2E}realax_{2E}real}). (\forall V1t \in (2^{ty_{2E}realax_{2E}real}). \\
& (\forall V2u \in (2^{ty_{2E}realax_{2E}real}). (((p (ap (ap (c_{2E}topology_{2E}Open_in \\
& ty_{2E}realax_{2E}real) (ap (ap (c_{2E}real_topology_{2E}subtopology \\
& ty_{2E}realax_{2E}real) c_{2E}real_topology_{2E}euclidean) V2u)) \\
& V0s)) \wedge (p (ap c_{2E}real_topology_{2E}Open V1t))) \Rightarrow (p (ap (ap (c_{2E}topology_{2E}Open_in \\
& ty_{2E}realax_{2E}real) (ap (ap (c_{2E}real_topology_{2E}subtopology \\
& ty_{2E}realax_{2E}real) c_{2E}real_topology_{2E}euclidean) V2u)) \\
& (ap (ap (c_{2E}pred_set_{2E}INTER ty_{2E}realax_{2E}real) V0s) V1t))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0s \in (2^{ty_{2E}realax_{2E}real}). (p (ap (ap (c_{2E}pred_set_{2E}SUBSET ty_{2E}realax_{2E}real) V0s) (ap c_{2E}real_topology_{2E}closure V0s)))) \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_{2E}realax_{2E}real}). (\forall V1t \in (2^{ty_{2E}realax_{2E}real}). \\
& ((p (ap (ap (c_{2E}pred_set_{2E}SUBSET ty_{2E}realax_{2E}real) V0s) \\
& V1t)) \Rightarrow (p (ap (ap (c_{2E}pred_set_{2E}SUBSET ty_{2E}realax_{2E}real) \\
& (ap c_{2E}real_topology_{2E}closure V0s)) (ap c_{2E}real_topology_{2E}closure \\
& V1t))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_{2E}realax_{2E}real}). (\forall V1t \in (2^{ty_{2E}realax_{2E}real}). \\
& (((p (ap (ap (c_{2E}pred_set_{2E}SUBSET ty_{2E}realax_{2E}real) V0s) \\
& V1t)) \wedge (p (ap c_{2E}real_topology_{2E}closed V1t))) \Rightarrow (p (ap (ap (c_{2E}pred_set_{2E}SUBSET \\
& ty_{2E}realax_{2E}real) (ap c_{2E}real_topology_{2E}closure V0s)) \\
& V1t))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_{2E}realax_{2E}real}). (\forall V1t \in (2^{ty_{2E}realax_{2E}real}). \\
& (((p (ap c_{2E}real_topology_{2E}bounded_def V1t)) \wedge (p (ap (ap (\\
& c_{2E}pred_set_{2E}SUBSET ty_{2E}realax_{2E}real) V0s) V1t))) \Rightarrow (p (\\
& ap c_{2E}real_topology_{2E}bounded_def V0s))))))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact V0s)) \Rightarrow (p (ap c_2Ereal_topology_2Ebounded_def V0s)))) \quad (50)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact V0s)) \Rightarrow (p (ap c_2Ereal_topology_2EClosed V0s)))) \quad (51)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact (ap c_2Ereal_topology_2Eclosure V0s))) \Leftrightarrow (p (ap c_2Ereal_topology_2Ebounded_def V0s)))) \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Elocally \\ & c_2Ereal_topology_2Ecompact) V0s)) \Leftrightarrow (\forall V1k \in (2^{ty_2Erealax_2Ereal}). \\ & (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V1k) \\ & V0s)) \wedge (p (ap c_2Ereal_topology_2Ecompact V1k))) \Rightarrow (\exists V2u \in \\ & (2^{ty_2Erealax_2Ereal}).(\exists V3v \in (2^{ty_2Erealax_2Ereal}). \\ & ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V1k) \\ & V2u)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\ & V2u) V3v)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\ & V3v) V0s)) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\ & (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\ & c_2Ereal_topology_2Eeuclidean) V0s)) V2u)) \wedge (p (ap c_2Ereal_topology_2Ecompact \\ & V3v)))))))))))))) \quad (53) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (63)$$

Theorem 1

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap c_2Ereal_topology_2Elocally \\ & c_2Ereal_topology_2Ecompact) V0s)) \Leftrightarrow (\forall V1k \in (2^{ty_2Erealax_2Ereal}). \\ & (\forall V2t \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V1k) V0s)) \wedge ((p (ap c_2Ereal_topology_2Ecompact \\ & V1k)) \wedge ((p (ap c_2Ereal_topology_2EOpen V2t)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V1k) V2t)))))) \Rightarrow (\exists V3u \in (2^{ty_2Erealax_2Ereal}). \\ & (\exists V4v \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V1k) V3u)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V3u) V4v)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V3u) V2t)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V4v) V0s)) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in \\ & ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\ & ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V0s)) \\ & V3u)) \wedge (p (ap c_2Ereal_topology_2Ecompact V4v)))))))))) \end{aligned}$$