

thm_2Ereal__topology_2ELOCALLY__COMPACT__DELETE
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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2F)$.

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_25C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 10 We define $c_2Ebool_2E_2F_2E_25C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (3)$$

Definition 12 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Epred_set_2EDIFF$

Definition 13 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EDELETE$

Definition 14 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap (c_2Eenum_2EREP_num$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (4)$$

Let $c_2Eenum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EREP_num \in (omega^{ty_2Eenum_2Eenum}) \quad (5)$$

Let $c_2Eenum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Eenum_2ESUC_REP \in (omega^{omega}) \quad (6)$$

Let $c_2Eenum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EABS_num \in (ty_2Eenum_2Eenum^{omega}) \quad (7)$$

Definition 15 We define c_2Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum. (ap c_2Eenum_2EABS_num$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum. (ap (c_2Eprim_rec_2E_3C$

Definition 19 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum. (ap (c_2Earithmetic_2E_3E$

Definition 20 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum. (ap (c_2Earithmetic_2E_3E_3D$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ereal_topology_2Enet A0) \quad (8)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Emk_net A_27a \in ((ty_2Ereal_topology_2Enet A_27a)^{(2^{A_27a})^{A_27a}}) \quad (9)$$

Definition 21 We define $c_2Ereal_topology_2Esequentially$ to be $(ap (c_2Ereal_topology_2Emk_net ty_2Ereal ty_2Erealax_2Ereal) : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (10)$$

Definition 22 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27a^{A_27c}). \lambda V2h \in (A_27b^{A_27c})$. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (11)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (13)$$

Definition 23 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E40 ty_2Ereal ty_2Erealax_2Ereal) : \iota$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Definition 24 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. \lambda V2T3 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)}) \quad (15)$$

Definition 25 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota. \lambda V0net \in (ty_2Ereal_topology_2Etrivial_limit\ A_27a)$

Definition 26 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1net \in (ty_2Ereal_topology_2Eeventually\ A_27a\ p)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (16)$$

Definition 27 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (17)$$

Definition 28 We define $c_2Ereal_topology_2E2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^{A_27a})$

Definition 29 We define $c_2Ereal_topology_2Ecompact$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_2Ebool_2E2\ ty_2Ereal ty_2Erealax_2Ereal) : \iota$

Definition 30 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$

Definition 31 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap$ (c_2Ebool_2E2

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (18)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (19)$$

Definition 32 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap$ ($c_2Etopology_2Etopology\ ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^A-27a)})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (20)$$

Definition 33 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap$ (c

Definition 34 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 35 We define $c_2Ereal_topology_2Elocally$ to be $\lambda V0P \in (2^{(ty_2Erealax_2Ereal)}).\lambda V1s \in (2^{ty_2Erealax$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(p\ (ap\ (ap\ (c_2Etopology_2Eopen_in \\ & ty_2Erealax_2Ereal)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\ & ty_2Erealax_2Ereal)\ c_2Ereal_topology_2Eeuclidean)\ V0s)) \\ & V0s))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned}
& (\forall V0u \in (2^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2a \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Etopology_2Eopen_in \\
& ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V0u)) \\
& V1s)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V0u)) (ap (ap (c_2Epred_set_2EDELETE \\
& ty_2Erealax_2Ereal) V1s) V2a)))))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V0s)) V1t)) \wedge (p (ap (ap c_2Ereal_topology_2Elocally \\
& c_2Ereal_topology_2Ecompact) V0s))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Elocally \\
& c_2Ereal_topology_2Ecompact) V1t))))))
\end{aligned} \tag{26}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1a \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_topology_2Elocally c_2Ereal_topology_2Ecompact) \\
& V0s)) \Rightarrow (p (ap (ap c_2Ereal_topology_2Elocally c_2Ereal_topology_2Ecompact) \\
& (ap (ap (c_2Epred_set_2EDELETE ty_2Erealax_2Ereal) V0s) V1a))))))
\end{aligned}$$