

thm_2Ereal__topology_2ELOCALLY__COMPACT__OPEN__IN (TMWq5kFcXh5qCzkZb5GVxwtU1SynsT7Es4i)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Definition 9 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40$ (

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \quad (7)$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (12)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Etopology_2Etopology\ A.27a \in ((ty_2Etopology_2Etopology\ A.27a)^{(2^{(2^A - 27a)}})) \quad (13)$$

Definition 16 We define $c_Ereal_topology_2Eeuclidean$ to be $(ap (c_Etopology_2Etopology ty_2Erealx$

Let $c_Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Etopology_2Eopen_in A_27a \in ((2^{(2^{A_27a})})^{(ty_2Etopology_2Etopology A_27a)}) \quad (14)$$

Let $c_Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (15)$$

Definition 17 We define $c_Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_$

Definition 18 We define $c_Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Let $c_Eenum_2EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_2EREP_num \in (\omega^{ty_2Eenum_2Eenum}) \quad (16)$$

Let $c_Eenum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 19 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum. (ap c_Eenum_2EABS_num$

Definition 20 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum$

Definition 21 We define $c_Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum$

Definition 22 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 23 We define $c_Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum. \lambda V1n \in ty_2Eenum_2Eenum$

Let $ty_2Ereal_topology_2Eenet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ereal_topology_2Eenet A0) \quad (18)$$

Let $c_Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ereal_topology_2Emk_net A_27a \in ((ty_2Ereal_topology_2Eenet A_27a)^{(2^{A_27a})^{A_27a}}) \quad (19)$$

Definition 24 We define $c_Ereal_topology_2Esequentially$ to be $(ap (c_Ereal_topology_2Emk_net ty_2E$

Definition 25 We define $c_Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in$

Let $c_Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ereal_topology_2Eenetord A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet A_27a)}) \quad (20)$$

Definition 26 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology$

Definition 27 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2$

Definition 28 We define $c_2Ereal_topology_2E2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 29 We define $c_2Ereal_topology_2Ecompact$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E$

Definition 30 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 31 We define $c_2Ereal_topology_2Ellocally$ to be $\lambda V0P \in (2^{(2^{ty_2Erealax_2Ereal})}).\lambda V1s \in (2^{ty_2Ere$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1u \in (2^{ty_2Erealax_2Ereal}). \\ & ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\ & (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\ & c_2Ereal_topology_2Eeuclidean) V1u)) V0s) \Leftrightarrow (\exists V2t \in (\\ & 2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Eopen V2t)) \wedge \\ & (V0s = (ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) V1u) \\ & V2t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Eopen \\ & V0s) \Rightarrow (p (ap (ap c_2Ereal_topology_2Ellocally c_2Ereal_topology_2Ecompact) \\ & V0s)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap c_2Ereal_topology_2Elocally c_2Ereal_topology_2Ecompact) \\
& V0s)) \wedge (p (ap (ap c_2Ereal_topology_2Elocally c_2Ereal_topology_2Ecompact) \\
& V1t))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Elocally c_2Ereal_topology_2Ecompact) \\
& (ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) V0s) V1t))))))
\end{aligned} \tag{27}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V0s)) V1t)) \wedge (p (ap (ap c_2Ereal_topology_2Elocally \\
& c_2Ereal_topology_2Ecompact) V0s))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Elocally \\
& c_2Ereal_topology_2Ecompact) V1t))))))
\end{aligned}$$