

thm_2Ereal__topology_2ELOCALLY__EMPTY (TMVQw86xm7hToBP1GKjDp5tgK7b2r5yww8s)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Eprod_set_2EEMPTY$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2E_2F)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Eprod_set_2EEMPTY$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal \tag{3}$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod ty_2Erealx_2Ereal ty_2Erealx_2Ereal)}) \tag{4}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{6}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{7}$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{8}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{9}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \tag{10}$$

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ap\ P\ x)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{11}$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 14 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (ap\ P\ x))$

Definition 15 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ (ap\ P\ x))))$

Definition 16 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E3F\ (ap\ P\ x)))$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \tag{12}$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \tag{13}$$

Definition 17 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in ((2^{(2^{A-27a})})^{(ty_2Etopology_2Etopology A_27a)}) \quad (14)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \quad (15)$$

Definition 18 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c_2E$

Definition 19 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 20 We define $c_2Ereal_topology_2Elocally$ to be $\lambda V0P \in (2^{(2^{ty_2Erealax_2Ereal})}). \lambda V1s \in (2^{ty_2Erealax_2Ereal})$

Definition 21 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E21 2) (\lambda V2t \in 2. (ap$

Definition 22 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E7E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (21) \end{aligned}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\exists V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A.27a. (p (ap V1Q V4x))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in A_27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b)^{A_27a}). (\forall V4x \in A_27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0u \in (2^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eeuclidean) V0u)) V1s)) \Leftrightarrow ((p (ap (ap (c_2Epred_set_2ESUBSET \\
& ty_2Erealax_2Ereal) V1s) V0u)) \wedge (\forall V2x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V2x) V1s)) \Rightarrow (\exists V3e \in \\
& ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V3e)) \wedge (\forall V4x_27 \in ty_2Erealax_2Ereal. ((\\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x_27) V0u)) \wedge (\\
& p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V4x_27) V2x))) V3e))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V4x_27) V1s))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge (\\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{48}$$

Theorem 1

$$\begin{aligned}
& (\forall V0P \in (2^{(2^{ty_2Erealax_2Ereal})}). (p \text{ (ap (ap c_2Ereal_topology_2Elocally} \\
& V0P) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))))
\end{aligned}$$