

thm\_2Ereal\_\_topology\_2ELOCALLY\_\_INJECTIVE\_\_LINEAR\_\_IMA  
(TMGsmfXurjWP-  
kkPLZ95VsbxYTiVdCDWMtxo)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 8** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A-27c}).\lambda V1g$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (4)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ t))$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (5)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2)))$

**Definition 16** We define  $c\_2Ereal\_topology\_2Elinear$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal}$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (9)$$

**Definition 17** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b))$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (10)$$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 19** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(\lambda p\ V1f\ V0x)))$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (12)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (13)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (14)$$

**Definition 20** We define  $c\_2Enum\_2E0$  to be  $(\lambda p\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 21** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$ .

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (16)$$

**Definition 22** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (A\_27b^{A\_27a})$ .

**Definition 23** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(\lambda p\ V1t\ V0s)$ .

**Definition 24** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(\lambda p\ (c\_2Ebool\_2EIN\ V0s))$ .

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (17)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (18)$$

**Definition 25** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap (c\_2Etopology\_2Etopology ty\_2Erealax$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology A\_27a)}) \quad (19)$$

**Definition 26** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2E$

**Definition 27** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 28** We define  $c\_2Ereal\_topology\_2Elocally$  to be  $\lambda V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})}). \lambda V1s \in (2^{ty\_2Erealax\_2Ereal})$

Let  $c\_2Ereal\_topology\_2Ehomeomorphism : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2Ehomeomorphism \in ((2^{(ty\_2Epair\_2Eprod (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) (ty\_2Erealax\_2Ereal))})^{(ty\_2Erealax\_2Ereal)}) \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg (p V0t)))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow (\neg(p\ V0A) \vee (p\ V1B)))))) \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (33)$$

Assume the following.

$$2. (((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in A\_27a. ((\forall V2x \in A\_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v)))))) \quad (35)$$

Assume the following.

$$(\forall V0r \in 2. (\forall V1p \in 2. (\forall V2q \in 2. (((((p\ V1p) \wedge (p\ V2q)) \Rightarrow (p\ V0r)) \Leftrightarrow ((p\ V1p) \Rightarrow ((p\ V2q) \Rightarrow (p\ V0r)))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). (\forall V2x \in A\_27c. ((ap\ (ap\ (ap\ (c.2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).((ap\ (ap \\ (c.2Epred\_set.2EIMAGE\ A.27a\ A.27a)\ (\lambda V1x \in A.27a.V1x))\ V0s) = \\ V0s)) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27c^{A.27b}).(\forall V1g \in (A.27b^{A.27a}). \\ (\forall V2s \in (2^{A.27a}).((ap\ (ap\ (c.2Epred\_set.2EIMAGE\ A.27a \\ A.27c)\ (ap\ (ap\ (c.2Ecombin.2Eo\ A.27a\ A.27c\ A.27b)\ V0f)\ V1g))\ V2s) = \\ (ap\ (ap\ (c.2Epred\_set.2EIMAGE\ A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c.2Epred\_set.2EIMAGE \\ A.27a\ A.27b)\ V1g)\ V2s)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in (2^{A.27a}).(\forall V1f \in (A.27a^{A.27b}).(\forall V2s \in \\ (2^{A.27b}).((\forall V3y \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V3y)\ (ap\ (ap\ (c.2Epred\_set.2EIMAGE\ A.27b\ A.27a)\ V1f)\ V2s))) \Rightarrow ( \\ p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(((p \\ (ap\ c.2Ereal\_topology.2Elinear\ V0f)) \wedge (\forall V1x \in ty.2Erealax.2Ereal. \\ (\forall V2y \in ty.2Erealax.2Ereal.(((ap\ V0f\ V1x) = (ap\ V0f\ V2y)) \Rightarrow \\ (V1x = V2y)))))) \Rightarrow (\exists V3g \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}). \\ ((p\ (ap\ c.2Ereal\_topology.2Elinear\ V3g)) \wedge ((ap\ (ap\ (c.2Ecombin.2Eo \\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal) \\ V3g)\ V0f) = (\lambda V4x \in ty.2Erealax.2Ereal.V4x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(\forall V1s \in \\ (2^{ty.2Erealax.2Ereal}).((p\ (ap\ c.2Ereal\_topology.2Elinear \\ V0f)) \Rightarrow (p\ (ap\ (ap\ c.2Ereal\_topology.2Econtinuous\_on\ V0f)\ V1s)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}), (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}), \\
& (\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}), (\forall V3g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}), ((p (ap (ap c\_2Ereal\_topology\_2Ehomeomorphism \\
& (ap (ap (c\_2Epair\_2E\_2C (2^{ty\_2Erealax\_2Ereal}) (2^{ty\_2Erealax\_2Ereal})) \\
& V0s) V1t)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})) V2f) V3g))) \Leftrightarrow ((\forall V4x \in \\
& ty\_2Erealax\_2Ereal, ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V4x) V0s)) \Rightarrow ((ap V3g (ap V2f V4x)) = V4x))) \wedge (((ap (ap (c\_2Epred\_set\_2EIMAGE \\
& ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V2f) V0s) = V1t) \wedge ((p ( \\
& ap (ap c\_2Ereal\_topology\_2Econtinuous\_on V2f) V0s)) \wedge ((\forall V5y \in \\
& ty\_2Erealax\_2Ereal, ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V5y) V1t)) \Rightarrow ((ap V2f (ap V3g V5y)) = V5y))) \wedge (((ap (ap (c\_2Epred\_set\_2EIMAGE \\
& ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V3g) V1t) = V0s) \wedge (p (ap \\
& (ap c\_2Ereal\_topology\_2Econtinuous\_on V3g) V1t))))))))))))) \\
& \tag{43}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})}), (\forall V1Q \in (2^{(2^{ty\_2Erealax\_2Ereal})}), \\
& (\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}), (\forall V3g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}), ((\forall V4s \in (2^{ty\_2Erealax\_2Ereal}), \\
& (\forall V5t \in (2^{ty\_2Erealax\_2Ereal}), ((p (ap (ap c\_2Ereal\_topology\_2Ehomeomorphism \\
& (ap (ap (c\_2Epair\_2E\_2C (2^{ty\_2Erealax\_2Ereal}) (2^{ty\_2Erealax\_2Ereal})) \\
& V4s) V5t)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})) V2f) V3g))) \Rightarrow ((p (ap \\
& V0P) V4s)) \Leftrightarrow (p (ap V1Q) V5t)))))) \Rightarrow (\forall V6s \in (2^{ty\_2Erealax\_2Ereal}), \\
& (\forall V7t \in (2^{ty\_2Erealax\_2Ereal}), ((p (ap (ap c\_2Ereal\_topology\_2Ehomeomorphism \\
& (ap (ap (c\_2Epair\_2E\_2C (2^{ty\_2Erealax\_2Ereal}) (2^{ty\_2Erealax\_2Ereal})) \\
& V6s) V7t)) (ap (ap (c\_2Epair\_2E\_2C (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}) \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})) V2f) V3g))) \Rightarrow ((p (ap \\
& (ap c\_2Ereal\_topology\_2Elocally V0P) V6s)) \Leftrightarrow (p (ap (ap c\_2Ereal\_topology\_2Elocally \\
& V1Q) V7t)))))))))) \\
& \tag{44}
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})})).(\forall V1Q \in (2^{(2^{ty\_2Erealax\_2Ereal})})). \\
& ((\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})).(\forall V3s \in \\
& (2^{ty\_2Erealax\_2Ereal})).(((p (ap c\_2Ereal\_topology\_2Elinear \\
& V2f)) \wedge (\forall V4x \in ty\_2Erealax\_2Ereal. (\forall V5y \in ty\_2Erealax\_2Ereal. \\
& (((ap V2f V4x) = (ap V2f V5y)) \Rightarrow (V4x = V5y)))))) \Rightarrow ((p (ap V0P (ap (ap ( \\
& c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V2f) V3s))) \Leftrightarrow (p (ap V1Q V3s)))))) \Rightarrow (\forall V6f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\
& (\forall V7s \in (2^{ty\_2Erealax\_2Ereal})).(((p (ap c\_2Ereal\_topology\_2Elinear \\
& V6f)) \wedge (\forall V8x \in ty\_2Erealax\_2Ereal. (\forall V9y \in ty\_2Erealax\_2Ereal. \\
& (((ap V6f V8x) = (ap V6f V9y)) \Rightarrow (V8x = V9y)))))) \Rightarrow ((p (ap (ap c\_2Ereal\_topology\_2Elocally \\
& V0P) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V6f) V7s))) \Leftrightarrow (p (ap (ap (ap c\_2Ereal\_topology\_2Elocally V1Q) V7s)))))))))
\end{aligned}$$