thm_2Ereal__topology_2ELOCALLY__TRANSLATION (TMR6Y4WNnZUrDvksCn8DVwnJFkmvKPXag3z)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 6 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 7 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 8 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27}$

Definition 9 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 10 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 13 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 14 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{9}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{10}$$

Definition 15 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{11}$$

Definition 16 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{12}$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{13}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{14}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{15}$$

Definition 17 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ t$

Definition 18 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 19 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 20 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 21 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 22 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{16}$$

Definition 23 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{17}$$

Definition 24 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 25 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{18}$$

Definition 26 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{19}$$

Definition 27 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 28 *We define* c_2Ereal__topology_2Econtinuous__on *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealc}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{20}$$

Definition 29 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 30 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 31 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E_2

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{21}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{22}$$

Definition 32 *We define* c_2Ereal__topology_2Eeuclidean *to be* $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax.$

Let $c\_2Etopology\_2Eopen\_\_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_\_in\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \tag{23}$$

Definition 33 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 34 *We define* c_2Ereal__topology_2Esubtopology *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Eto$

Definition 35 *We define* c_2Ereal__topology_2Elocally *to be* $\lambda V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})}).\lambda V1s \in (2^{ty\_2Ere$

Let $c\_2Ereal\_\_topology\_2Ehomeomorphism : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Ehomeomorphism \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Ereal \tag{24}$$

Definition 36 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{29}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{36}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B))))) \tag{40}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{41}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \tag{43}$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.((((p\ V1p)\wedge (p\ V2q))\Rightarrow(p\ V0r))\Leftrightarrow((p\ V1p)\Rightarrow((p\ V2q)\Rightarrow(p\ V0r))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27b)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27b))\ V0f) = V0f)\wedge((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)) = V0f))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27a)\ (\lambda V1x \in A\_27a.V1x))\ V0s) = V0s)) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow(\forall V0f \in (A\_27c^{A\_27b}).(\forall V1g \in (A\_27b^{A\_27a}). (\forall V2s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ V1g))\ V2s) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1g)\ V2s)))))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V0x)))) \tag{49}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) V0x) = V0x)) \tag{52}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x)) \tag{53}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y))))) \tag{55}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{56}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V0x))) \tag{57}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x))) \Leftrightarrow (V0x = V1y)))) \tag{58}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V1y) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))))) \tag{59}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1x)\ V0y))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0y)\ V1x))))))\tag{60}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V1y))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))))))\tag{61}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x)))))\tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = V0x))\tag{63}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))))\tag{64}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z))))))\tag{65}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1c \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ (\lambda V2x \in ty\_2Erealax\_2Ereal.V1c))\ V0s))))\tag{66}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V0f)\ V2s))\land (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V1g)\ V2s)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ (\lambda V3x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x))))\ V2s)))))) \tag{67}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ (\lambda V1x \in ty\_2Erealax\_2Ereal.V1x))\ V0s))) \tag{68}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V3g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ c\_2Ereal\_\_topology\_2Ehomeomorphism\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ V2f)\ V3g)))\Leftrightarrow((\forall V4x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4x)\ V0s))\Rightarrow((ap\ V3g\ (ap\ V2f\ V4x)) = V4x)))\land(((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V2f)\ V0s) = V1t)\land((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V2f)\ V0s))\land((\forall V5y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V5y)\ V1t))\Rightarrow((ap\ V2f\ (ap\ V3g\ V5y)) = V5y)))\land(((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V3g)\ V1t) = V0s)\land(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econtinuous\_\_on}\ V3g)\ V1t)))))))))))) \tag{69}$$

Assume the following.

$$(\forall V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})}).(\forall V1Q \in (2^{(2^{ty\_2Erealax\_2Ereal})}).(\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V3g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((\forall V4s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V5t \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ c\_2Ereal\_\_topology\_2Ehomeomorphism\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V4s)\ V5t))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ V2f)\ V3g)))\Rightarrow((p\ (ap\ V0P\ V4s))\Leftrightarrow(p\ (ap\ V1Q\ V5t))))))\Rightarrow(\forall V6s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V7t \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ c\_2Ereal\_\_topology\_2Ehomeomorphism\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V6s)\ V7t))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}))\ V2f)\ V3g)))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Elocally}\ V0P)\ V6s))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Elocally}\ V1Q)\ V7t))))))))))) \tag{70}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{73}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{82}$$

Theorem 1

$$(\forall V0P \in (2^{(2^{ty\_2Erealax\_2Ereal})}).((\forall V1a \in ty\_2Erealax\_2Ereal.\ (\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V3x \in ty\_2Erealax\_2Ereal.\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1a)\ V3x)))\ V2s)))\Leftrightarrow(p\ (ap\ V0P\ V2s)))))\Rightarrow (\forall V4a \in ty\_2Erealax\_2Ereal.(\forall V5s \in (2^{ty\_2Erealax\_2Ereal}).\ ((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Elocally}\ V0P)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V6x \in ty\_2Erealax\_2Ereal.\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V4a)\ V6x)))\ V5s)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Elocally}\ V0P)\ V5s)))))))$$