

thm_2Ereal__topology_2EMIDPOINT__IN__SEGMENT
 (TMbKbwjte-
 QUC61VAoBoqmS4xkcw8XquNcHv)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (5)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 8 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(a$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum}) \quad (14)$$

Definition 12 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Let $c_2Ereal_topology_2Emidpoint : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Emidpoint \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (15)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (16)$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2Eopen_segment : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eopen_segment \in ((2^{(ty_2Erealax_2Ereal)})_{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (17)$$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 14 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_lt)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 15 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 16 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 18 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (20)$$

Definition 20 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 21 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 22 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 23 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 24 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (21)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (22)$$

Definition 25 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 26 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (24)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a)^{(ty_2Elist_2Elist A_27a)} \quad (25)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (26)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (27)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (28)$$

Definition 27 We define $c_Ereal_topology_Eclosed_segment$ to be $\lambda V0l \in (ty_Elist_Elist (ty_Epair$

Definition 28 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda p V1f V0x))$

Definition 29 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\lambda p (c_Ebool_E21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \tag{34}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{35}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \tag{36}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{37}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
V0x) V2z))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Einv V0x) = \\
& (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x)))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Ereal_2E_2F V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) (ap (ap \\
& c_2Ereal_2E_2F V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) = V0x))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((ap (ap c_2Ereal_2Ereal_sub \\
V0x) V1y) = V2z) \Leftrightarrow (V0x = (ap (ap c_2Erealax_2Ereal_add V2z) V1y))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& \quad \quad ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) \wedge (p (ap \\
& (ap c_2Erealax_2Ereal_lt (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& \quad \quad ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& \quad \quad ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) \wedge (p (ap \\
& (ap c_2Ereal_2Ereal_lte (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& \quad \quad ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))))))))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_topology_2Emidpoint (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) V0a) V1b)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Erealax_2Einv (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))))) (ap (ap \\
& \quad \quad c_2Erealax_2Ereal_add V0a) V1b)))))) \\
& \hspace{15em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealx_2Ereal. (\forall V1b \in ty_2Erealx_2Ereal. \\
& \quad (\forall V2x \in ty_2Erealx_2Ereal. (((p (ap (ap (c_2Ebool_2EIN \\
& \quad ty_2Erealx_2Ereal) V2x) (ap c_2Ereal_topology_2Eclosed_segment \\
& \quad (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealx_2Ereal \\
& \quad ty_2Erealx_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealx_2Ereal \\
& \quad ty_2Erealx_2Ereal) V0a) V1b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& \quad ty_2Erealx_2Ereal ty_2Erealx_2Ereal)))))) \Leftrightarrow (\exists V3u \in \\
& \quad ty_2Erealx_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V3u)) \wedge ((p (ap (ap c_2Ereal_2Ereal_lte V3u) (ap \\
& \quad c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))))) \wedge (V2x = (ap (ap c_2Erealx_2Ereal_add \\
& \quad (ap (ap c_2Erealx_2Ereal_mul (ap (ap c_2Ereal_2Ereal_sub (\\
& \quad ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap \\
& \quad c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V3u)) V0a)) \\
& \quad (ap (ap c_2Erealx_2Ereal_mul V3u) V1b)))))) \wedge ((p (ap (ap (c_2Ebool_2EIN \\
& \quad ty_2Erealx_2Ereal) V2x) (ap c_2Ereal_topology_2Eopen_segment \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealx_2Ereal ty_2Erealx_2Ereal) \\
& \quad V0a) V1b)))) \Leftrightarrow ((\neg(V0a = V1b)) \wedge (\exists V4u \in ty_2Erealx_2Ereal. \\
& \quad ((p (ap (ap c_2Erealx_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) V4u)) \wedge ((p (ap (ap c_2Erealx_2Ereal_lt V4u) (ap \\
& \quad c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))))) \wedge (V2x = (ap (ap c_2Erealx_2Ereal_add \\
& \quad (ap (ap c_2Erealx_2Ereal_mul (ap (ap c_2Ereal_2Ereal_sub (\\
& \quad ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap \\
& \quad c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V4u)) V0a)) \\
& \quad (ap (ap c_2Erealx_2Ereal_mul V4u) V1b)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{61}$$

Theorem 1

$$\begin{aligned} & ((\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\ & (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Emidpoint \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V0a) V1b))) (ap c_2Ereal_topology_2Eclosed_segment (ap (ap \\ & (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)) \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V0a) V1b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal)))))) \wedge (\forall V2a \in ty_2Erealax_2Ereal. \\ & (\forall V3b \in ty_2Erealax_2Ereal. (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ & (ap c_2Ereal_topology_2Emidpoint (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V2a) V3b))) (ap c_2Ereal_topology_2Eopen_segment \\ & (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ & V2a) V3b)))) \Leftrightarrow (\neg (V2a = V3b)))))) \end{aligned}$$