thm_2Ereal__topology_2ENET (TMame7NA5a5LXpzQa25o1dNeBcmmwpgy6Sy)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Ereal\_\_topology\_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_\_topology\_2Enet\ A0) \tag{1}$$

Let $c\_2Ereal\_\_topology\_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_\_topology\_2Enet\ A\_27a)}) \tag{2}$$

Let $c\_2Ereal\_\_topology\_2Emk\_\_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a \in ((ty\_2Ereal\_\_topology\_2Enet\ A\_27a)^{((2^{A\_27a})^{A\_27a})}) \tag{3}$$

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0a \in (ty\_2Ereal\_\_topology\_2Enet\ A\_27a).((ap\ (c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a)\ (ap\ (c\_2Ereal\_\_topology\_2Enetord\ A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in ((2^{A\_27b})^{A\_27b}).((\forall V2x \in A\_27b.(\forall V3y \in A\_27b.((\forall V4z \in A\_27b.((p\ (ap\ (ap\ V1r\ V4z)\ V2x)) \Rightarrow (p\ (ap\ (ap\ V1r\ V4z)\ V3y)))) \vee (\forall V5z \in A\_27b.((p\ (ap\ (ap\ V1r\ V5z)\ V3y)) \Rightarrow (p\ (ap\ (ap\ V1r\ V5z)\ V2x))))))) \Leftrightarrow ((ap\ (c\_2Ereal\_\_topology\_2Enetord\ A\_27b)\ (ap\ (c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27b)\ V1r)) = V1r)))) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in (ty\_2Ereal\_\_topology\_2Enet\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((\forall V3z \in A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Ereal\_\_topology\_2Enetord\ A\_27a)\ V0n)\ V3z)\ V1x)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_\_topology\_2Enetord\ A\_27a)\ V0n)\ V3z)\ V2y)))) \vee (\forall V4z \in A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Ereal\_\_topology\_2Enetord\ A\_27a)\ V0n)\ V4z)\ V2y)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_\_topology\_2Enetord\ A\_27a)\ V0n)\ V4z)\ V1x))))))))$$