thm_2Ereal__topology_2EOPEN__IN__CONNECTED__COMPONE (TMFonzF839UiVdjucdt2X1tf5VEHCFLxL8L)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 5 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 6 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 7 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 8 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 9 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27a}$

Definition 10 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 15 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{4}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 16 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 17 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{6}$$

Let $c\_2Etopology\_2Eopen\_\_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_\_in\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \tag{7}$$

Definition 18 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 19 *We define* c_2Etopology_2Etopspace *to be* $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology\ $

Definition 20 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 21 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{8}$$

Definition 22 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 23 *We define* $\mathsf{c\_2Epred\_\_set\_2EEMPTY}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 24 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 25 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 26 *We define* $\mathsf{c\_2Epred\_\_set\_2EUNION}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{9}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{10}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{11}$$

Definition 27 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_REP}$ *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{12}$$

Definition 28 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_lt}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{13}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{14}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{15}$$

Definition 29 *We define* $\mathsf{c\_2Enum\_2E0}$ *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 30 *We define* $\mathsf{c\_2Ereal\_\_topology\_2EOpen}$ *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (\mathsf{c\_2Ebool\_2E\_2}$

Definition 31 *We define* c_2Ereal__topology_2Econnected *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ c_2Ebool_2

Definition 32 *We define* c_2Ereal__topology_2Econnected__component *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).\lambda V$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{17}$$

Definition 33 *We define* c_2Ereal__topology_2Eeuclidean *to be* $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Definition 34 *We define* c_2Ereal__topology_2Esubtopology *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Et$

Definition 35 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 36 *We define* c_2Etopology_2Eclosed__in *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Definition 37 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 38 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge((\neg False)\Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x))\vee(p\ V0Q)))\Leftrightarrow((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x)))\vee(p\ V0Q))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P)\vee(p\ (ap\ V1Q\ V2x))))\Leftrightarrow((p\ V0P)\vee(\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{33}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B)\wedge (p\ V2C))\vee(p\ V0A))\Leftrightarrow(((p\ V1B)\vee(p\ V0A))\wedge((p\ V2C)\vee(p\ V0A))))))) \tag{34}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{35}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27b)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27b))\ V0f) = V0f)\wedge((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)) = V0f))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t)\Leftrightarrow(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ A\_27a)\ V0s)\ V1t)))\Leftrightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1f \in (A\_27b^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V1f)\ V0s)))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow (\forall V0Q \in (2^{A\_27b}).((\forall V1P \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}).((\forall V3z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V3z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ A\_27a)\ (\lambda V4x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A\_27a.((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q\ (ap\ V2f\ V5x)))))))) \wedge ((\forall V6P \in ((2^{A\_27d})^{A\_27c}).(\forall V7f \in ((A\_27b^{A\_27d})^{A\_27c}).((\forall V8z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V8z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27d))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27c\ A\_27d\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V9x \in A\_27c.(\lambda V10y \in A\_27d.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))))) \Rightarrow (p\ (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A\_27c.(\forall V12y \in A\_27d.((p\ (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y))))))))) \wedge (\forall V13P \in (((2^{A\_27g})^{A\_27f})^{A\_27e}).(\forall V14f \in (((A\_27b^{A\_27g})^{A\_27f})^{A\_27e}).((\forall V15z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V15z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V16w \in A\_27e.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27f\ A\_27g\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V17x \in A\_27f.(\lambda V18y \in A\_27g.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x)\ V18y)))))))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A\_27e.(\forall V20x \in A\_27f.(\forall V21y \in A\_27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1u \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ A\_27a)\ V0top)\ V1u))\ V1u)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1u)\ (ap\ (\mathsf{c\_2Etopology\_2Etopspace}\ A\_27a)\ V0top)))))) \tag{46}$$

Assume the following.

$$((ap\ (\mathsf{c\_2Etopology\_2Etopspace}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean}) = (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Erealax\_2Ereal)) \tag{47}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V1x))\ V0s)))) \tag{48}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((\neg((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V1a))\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V2b)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V1a)\ V0s)) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V2b)\ V0s)) \land ((ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V1a) = (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V2b)))))))) \tag{49}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (2^{ty\_2Erealax\_2Ereal})\ ty\_2Erealax\_2Ereal)\ (\lambda V1x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ 2)\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V1x))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V1x)\ V0s))))) = V0s)) \tag{50}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V1x))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{(2^{A\_27a})}).(\forall V1t \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ V0s))\ V1t) = (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (2^{A\_27a})\ (2^{A\_27a}))\ (\lambda V2x \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V2x)\ V1t))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2x)\ V0s))))))))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor ((p\ V1q) \lor (p\ V2r))) \land (((p\ V0p) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V1q)))) \land (((p\ V1q) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V0p)))) \land ((p\ V2r) \lor ((\neg(p\ V1q)) \lor (\neg(p\ V0p))))))))))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \land (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor ((\neg(p\ V1q)) \lor (\neg(p\ V2r)))) \land (((p\ V1q) \lor (\neg(p\ V0p))) \land ((p\ V2r) \lor (\neg(p\ V0p))))))))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \lor (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor (\neg(p\ V1q))) \land (((p\ V0p) \lor (\neg(p\ V2r))) \land ((p\ V1q) \lor ((p\ V2r) \lor (\neg(p\ V0p)))))))))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ V1s))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ V2t)))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V1s)\ V2t))))))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1s \in (2^{(2^{A\_27a})}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ (2^{A\_27a}))\ V1s))\wedge(\forall V2t \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (2^{A\_27a}))\ V2t)\ V1s))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ V2t)))))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIGUNION}\ A\_27a)\ V1s)))))) \tag{70}$$

Theorem 1

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1x \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (2^{ty\_2Erealax\_2Ereal})\ ty\_2Erealax\_2Ereal)\ (\lambda V2x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ 2)\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V2x))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V2x)\ V0s))))))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ (ap\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Econnected\_\_component}\ V0s)\ V1x))))))$$