

thm_2Ereal__topology_2EOPEN__IN__SUBTOPOLOGY__UNION
(TML-
GrAZGLzoUwpV6XnhLPyKwyZm8K63ACDE)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 9 We define `c_2Epred_set_2EUNION` to be $\lambda A_{.27a} : \iota. \lambda V0s \in (2^{A_{.27a}}). \lambda V1t \in (2^{A_{.27a}}). (ap (c_{.27a} _2Epred_set_2EUNION) V0s V1t)$

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A_{.27a} : \iota. (\lambda V0P \in (2^{A_{.27a}}). (ap V0P (ap (c_{.27a} _2Emin_2E_40) V0P)))$

Let `ty_2Etopology_2Etopology` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Etopology_2Etopology A0) \quad (4)$$

Let `c_2Etopology_2Eopen_in` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow c_2Etopology_2Eopen_in A_{.27a} \in ((2^{(2^{A_{.27a}})}) (ty_2Etopology_2Etopology A_{.27a})) \quad (5)$$

Definition 12 We define `c_2Epred_set_2EINTER` to be $\lambda A_{.27a} : \iota. \lambda V0s \in (2^{A_{.27a}}). \lambda V1t \in (2^{A_{.27a}}). (ap (c_{.27a} _2Epred_set_2EINTER) V0s V1t)$

Let `c_2Etopology_2Etopology` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow c_2Etopology_2Etopology A_{.27a} \in ((ty_2Etopology_2Etopology A_{.27a})^{(2^{(2^{A_{.27a}})})}) \quad (6)$$

Definition 13 We define `c_2Ereal_topology_2Esubtopology` to be $\lambda A_{.27a} : \iota. \lambda V0top \in (ty_2Etopology_2Etopology A_{.27a})$

Definition 14 We define `c_2Ebool_2E_2F` to be $(ap (c_{.27a} _2Ebool_2E_2F) (\lambda V0t \in 2. V0t))$.

Definition 15 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (ap (ap c_{.27a} _2Emin_2E_3D_3D_3E V0t) c_{.27a} _2Ebool_2E_7E) V0t))$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (12)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p (ap (ap (c_2Ebool_2EIN A.27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A.27a) V2x) V1t)))))) \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (21) \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ V2x)\ (ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (22) \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0top \in (ty.2Etopology.2Etopology \\ A.27a). (\forall V1u \in (2^{A.27a}). (\forall V2s \in (2^{A.27a}). ((p\ (\\ ap\ (ap\ (c.2Etopology.2Eopen_in\ A.27a)\ (ap\ (ap\ (c.2Ereal_topology.2Esubtopology \\ A.27a)\ V0top)\ V1u))\ V2s)) \Leftrightarrow (\exists V3t \in (2^{A.27a}). ((p\ (ap\ (ap\ (\\ c.2Etopology.2Eopen_in\ A.27a)\ V0top)\ V3t)) \wedge (V2s = (ap\ (ap\ (c.2Epred_set.2EINTER \\ A.27a)\ V3t)\ V1u)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False)) \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\
& \ A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). (((p \\
& (ap (ap (c_2Etopology_2Eopen_in \ A_27a) \ V0top) \ V1s)) \wedge (p (ap (ap \\
& (c_2Etopology_2Eopen_in \ A_27a) \ V0top) \ V2t))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\
& \ A_27a) \ V0top) (ap (ap (c_2Epred_set_2EINTER \ A_27a) \ V1s) \ V2t))))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\
& \ A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). (\forall V3u \in \\
& (2^{A_27a}). (((p (ap (ap (c_2Etopology_2Eopen_in \ A_27a) (ap (ap \\
& (c_2Ereal_topology_2Esubtopology \ A_27a) \ V0top) \ V2t)) \ V1s)) \wedge \\
& (p (ap (ap (c_2Etopology_2Eopen_in \ A_27a) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \ A_27a) \ V0top) \ V3u)) \ V1s))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\
& \ A_27a) (ap (ap (c_2Ereal_topology_2Esubtopology \ A_27a) \ V0top) \\
& (ap (ap (c_2Epred_set_2EUNION \ A_27a) \ V2t) \ V3u)) \ V1s))))))
\end{aligned}$$