thm_2Ereal__topology_2EOPEN__MAP__CLOSED__SUPERSET__P (TMXXe-suUJ3JdxpHugQdRQ4Qu9Zv1ZUknPj9)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 5 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 10 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 11 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{5}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{7}$$

Definition 12 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{8}$$

Definition 13 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 15 *We define* c_2Ereal__topology_2EOpen *to be* $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Ebool_2E_2

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{13}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{14}$$

Definition 16 *We define* c_2Ereal__topology_2Eeuclidean *to be* $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Let $c\_2Etopology\_2Eopen\_\_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_\_in\ A\_27a \in ((2^{(2^{A\_27a})})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \tag{15}$$

Definition 17 *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 18 *We define* c_2Ereal__topology_2Esubtopology *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Eto$

Definition 19 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 20 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 21 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 22 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 23 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 24 *We define* c_2Etopology_2Etopspace *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Definition 25 *We define* c_2Etopology_2Eclosed__in *to be* $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{20}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{26}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{27}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{28}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{29}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a)\wedge(p\ (ap\ V0P\ V2x))))\Leftrightarrow(p\ (ap\ V0P\ V1a))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b))\Leftrightarrow((V0x = V2a)\wedge(V1y = V3b))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t)\Leftrightarrow(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f)))\Leftrightarrow(\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V0s)\ V1t)))\Leftrightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s))\wedge(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ A\_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V2t)\ V1s)))))) \tag{37}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ V0s))) \tag{38}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V0s))\ V0s))) \tag{39}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}).(\forall V3u \in (2^{ty\_2Erealax\_2Ereal}).(\forall V4w \in (2^{ty\_2Erealax\_2Ereal}).(((\forall V5k \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V5k))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V5k)))))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V3u))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V4w)\ V2t))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V6x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V6x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V6x)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V6x))\ V4w))))))\ V3u)))))\Rightarrow(\exists V7v \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ V7v))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ V4w)\ V7v))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V8x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V8x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V8x)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V8x))\ V7v))))))\ V3u)))))))))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{52}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{53}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ V1s))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ V2t)))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V1s)\ V2t))))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0top \in (ty\_2Etopology\_2Etopology\ A\_27a).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ V1s))\wedge(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ A\_27a)\ V0top)\ V2t)))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ A\_27a)\ V0top)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDIFF}\ A\_27a)\ V1s)\ V2t))))))) \tag{57}$$

Theorem 1

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) V0f)\ V1s))\ V2t))\Rightarrow((\forall V3k \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean}) V1s))\ V3k))\Rightarrow(p\ (ap\ (ap\ (c\_2Etopology\_2Eopen\_\_in\ ty\_2Erealax\_2Ereal) (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal) \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE} ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V3k)))))\Leftrightarrow(\forall V4u \in (2^{ty\_2Erealax\_2Ereal}).(\forall V5w \in (2^{ty\_2Erealax\_2Ereal}). (((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in}\ ty\_2Erealax\_2Ereal) (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology}\ ty\_2Erealax\_2Ereal) \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V1s))\ V4u))\land((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET} ty\_2Erealax\_2Ereal)\ V5w)\ V2t))\land(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET} ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)\ (\lambda V6x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} ty\_2Erealax\_2Ereal\ 2)\ V6x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V6x)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN} ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V6x))\ V5w))))))\ V4u))))\Rightarrow(\exists V7v \in (2^{ty\_2Erealax\_2Ereal}).((p\ (ap\ (ap\ (\mathsf{c\_2Etopology\_2Eclosed\_\_in} ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2Esubtopology} ty\_2Erealax\_2Ereal)\ \mathsf{c\_2Ereal\_\_topology\_2Eeuclidean})\ V2t)) V7v))\land((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal) V5w)\ V7v))\land(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ ty\_2Erealax\_2Ereal) (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) (\lambda V8x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ V8x)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal) V8x)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V8x)) V7v))))))\ V4u)))))))))))))$$