

thm\_2Ereal\_\_topology\_2EOPEN\_\_MAP\_\_IFF\_\_LOWER\_\_HEMICON  
(TM-  
bAHXGXXqju6UVjEXEz3khNFgLUvcsoxCY)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 6** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ebool\_2E\_2F\_5C$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (3)$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda P \in (2^{A \cdot 27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 9** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.\lambda P \in (2^{A \cdot 27a}).(ap V1f \in (2^{A \cdot 27a}).(ap V1f V0x))$

**Definition 10** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A.\lambda P \in (A \cdot 27b^{A \cdot 27a}).\lambda V1s \in$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (7)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 ($

Let  $c\_2Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 14** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (13)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

**Definition 15** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap (c\_2Etopology\_2Etopology ty\_2Erealax\_2Ereal$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in A\_27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology A\_27a)}) \quad (15)$$

**Definition 16** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Ebool\_2E2$

**Definition 17** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 18** We define  $c\_2Ebool\_2E2F$  to be  $(ap (c\_2Ebool\_2E21\_2) (\lambda V0t \in 2.V0t))$ .

**Definition 19** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E21\_2) (\lambda V2t \in 2.V2t))))$

**Definition 20** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E3D\_3D\_3E V0t) c\_2Ebool\_2E2F))$

**Definition 21** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^A-27a)}). (ap (c\_2Epred\_set\_2E2$

**Definition 22** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 23** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Ebool\_2E2$

**Definition 24** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c\_2Ebool\_2E2$

**Definition 25** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ & 2^{A\_27a}).((\forall V2x \in A\_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & V0P) \vee (\forall V3x \in A\_27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg( \\ & p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \end{aligned} \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (29)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V1t))))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2E}pair_{.2E}prod A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V1v) (ap (c_{.2E}pred_{.set}_{.2E}GSPEC A_{.27a} A_{.27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} 2) V1v) c_{.2E}bool_{.2E}ET) = (ap V0f V2x))))))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) (ap (ap (c_{.2E}pred_{.set}_{.2E}DIFF A_{.27a}) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V0s)) \wedge (\neg (p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V1t)))))))))) \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\ & \quad \forall V0y \in A_{.27b}.(\forall V1s \in (2^{A_{.27a}}).(\forall V2f \in (A_{.27b}^{A_{.27a}}). \\ & ((p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27b})\ V0y)\ (ap\ (ap\ (c_{.2Epred\_set\_2EIMAGE} \\ & \quad A_{.27a}\ A_{.27b})\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_{.27a}.((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_{.2Ebool\_2EIN}\ A_{.27a})\ V3x)\ V1s))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \Rightarrow ((p\ V0P) \Rightarrow (p\ V1Q)))))) \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0top \in (ty_{.2Etopology\_2Etopology} \\ & \quad A_{.27a}).(\forall V1s \in (2^{A_{.27a}}).(\forall V2t \in (2^{A_{.27a}}).((p\ ( \\ & ap\ (ap\ (c_{.2Etopology\_2Eopen\_in}\ A_{.27a})\ (ap\ (ap\ (c_{.2Ereal\_topology\_2Esubtopology} \\ & \quad A_{.27a})\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_{.2Epred\_set\_2ESUBSET}\ A_{.27a}) \\ & \quad \quad V2t)\ V1s))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0top \in (ty_{.2Etopology\_2Etopology} \\ & \quad A_{.27a}).(\forall V1s \in (2^{A_{.27a}}).(\forall V2t \in (2^{A_{.27a}}).((p\ ( \\ & ap\ (ap\ (c_{.2Etopology\_2Eclosed\_in}\ A_{.27a})\ (ap\ (ap\ (c_{.2Ereal\_topology\_2Esubtopology} \\ & \quad A_{.27a})\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_{.2Epred\_set\_2ESUBSET}\ A_{.27a}) \\ & \quad \quad V2t)\ V1s))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0s \in (2^{ty_{.2Erealax\_2Ereal}}).((ap\ (c_{.2Etopology\_2Etopspace}\ ty_{.2Erealax\_2Ereal})\ (ap\ (ap\ (c_{.2Ereal\_topology\_2Esubtopology}\ ty_{.2Erealax\_2Ereal})\ c_{.2Ereal\_topology\_2Euclidean})\ V0s)) = V0s)) \quad (40)$$

Assume the following.

$$(\forall V0s \in (2^{ty_{.2Erealax\_2Ereal}}).((p\ (ap\ (ap\ (c_{.2Etopology\_2Eopen\_in}\ ty_{.2Erealax\_2Ereal})\ (ap\ (ap\ (c_{.2Ereal\_topology\_2Esubtopology}\ ty_{.2Erealax\_2Ereal})\ c_{.2Ereal\_topology\_2Euclidean})\ V0s))\ V0s))) \quad (41)$$

Assume the following.

$$(\forall V0s \in (2^{ty_{.2Erealax\_2Ereal}}).((p\ (ap\ (ap\ (c_{.2Etopology\_2Eclosed\_in}\ ty_{.2Erealax\_2Ereal})\ (ap\ (ap\ (c_{.2Ereal\_topology\_2Esubtopology}\ ty_{.2Erealax\_2Ereal})\ c_{.2Ereal\_topology\_2Euclidean})\ V0s))\ V0s))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A.27a).(\forall V1s \in (2^{A.27a}).((p \ (ap \ (ap \ (c\_2Etopology\_2Eopen\_in \\ A.27a) \ V0top) \ V1s)) \Leftrightarrow ((p \ (ap \ (ap \ (c\_2Epred\_set\_2ESUBSET \ A.27a) \\ V1s) \ (ap \ (c\_2Etopology\_2Etopospace \ A.27a) \ V0top))) \wedge (p \ (ap \ (ap \ ( \\ c\_2Etopology\_2Eclosed\_in \ A.27a) \ V0top) \ (ap \ (ap \ (c\_2Epred\_set\_2EDIFF \\ A.27a) \ (ap \ (c\_2Etopology\_2Etopospace \ A.27a) \ V0top)) \ V1s)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A.27a).(\forall V1s \in (2^{A.27a}).(\forall V2t \in (2^{A.27a}).(((p \\ (ap \ (ap \ (c\_2Etopology\_2Eopen\_in \ A.27a) \ V0top) \ V1s)) \wedge (p \ (ap \ (ap \\ (c\_2Etopology\_2Eclosed\_in \ A.27a) \ V0top) \ V2t))) \Rightarrow (p \ (ap \ (ap \ (c\_2Etopology\_2Eopen\_in \\ A.27a) \ V0top) \ (ap \ (ap \ (c\_2Epred\_set\_2EDIFF \ A.27a) \ V1s) \ V2t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ A.27a).(\forall V1s \in (2^{A.27a}).(\forall V2t \in (2^{A.27a}).(((p \\ (ap \ (ap \ (c\_2Etopology\_2Eclosed\_in \ A.27a) \ V0top) \ V1s)) \wedge (p \ (ap \\ (ap \ (c\_2Etopology\_2Eopen\_in \ A.27a) \ V0top) \ V2t))) \Rightarrow (p \ (ap \ (ap \ ( \\ c\_2Etopology\_2Eclosed\_in \ A.27a) \ V0top) \ (ap \ (ap \ (c\_2Epred\_set\_2EDIFF \\ A.27a) \ V1s) \ V2t)))))) \end{aligned} \quad (60)$$



**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap ( \\
& \quad ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V0f) V1s)) V2t)) \Rightarrow ((\forall V3u \in (2^{ty\_2Erealax\_2Ereal}).((p ( \\
& \quad ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap (ap ( \\
& \quad c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) \\
& \quad V1s)) V3u)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V3u)))))) \Leftrightarrow (\forall V4u \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Etopology\_2Eclosed\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V1s)) \\
& \quad V4u)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V5y \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V5y) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5y) V2t)) (ap (ap ( \\
& \quad c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V6x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V6x) V1s)) (ap (ap ( \\
& \quad c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) (ap V0f V6x)) V5y)))))) V4u)))))))))
\end{aligned}$$