

thm_2Ereal_topology_2EPASTING_LEMMA (TMP6sxtxZyoNzo2T2T3jy2bbXH39Lgsofjg)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 7 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 9 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27a^{A_27c}).$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A.\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (3)$$

Definition 14 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 15 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 ($

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 16 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 18 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 19 We define $c_Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap\ (c_2Ebool_2E_2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

Definition 20 We define $c_Ereal_topology_2Eeuclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^A-27a)})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (15)$$

Definition 21 We define $c_Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 22 We define $c_Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (16)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (17)$$

Definition 23 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A-27$

Definition 24 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 25 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. V2t))$

Definition 26 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ (c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 27 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^A-27a)}). (ap\ (c_2Epred_set_2E$

Definition 28 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 29 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ ($

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). (((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (42)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (43)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}})^{A_{27a}}).((\forall V1x \in A_{27a}.(\exists V2y \in A_{27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}).(\forall V4x \in A_{27a}.(p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (44)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c_{2Ecombin_2EI} A_{27a}) V0x) = V0x)) \quad (45)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}).(((ap (ap (c_{2Ecombin_2Eo} A_{27a} A_{27b}) (c_{2Ecombin_2EI} A_{27b})) V0f) = V0f) \wedge ((ap (ap (c_{2Ecombin_2Eo} A_{27a} A_{27b} A_{27a}) V0f) (c_{2Ecombin_2EI} A_{27a})) = V0f))) \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{10em} (47) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \\ & \hspace{10em} (48) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \hspace{10em} (49) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ \\ & \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V1t)))))) \\ & \hspace{10em} (50) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& (\forall V0Q \in (2^{A.27b}).(\forall V1P \in (2^{A.27a}).(\forall V2f \in \\
& (A.27b^{A.27a}).(\forall V3z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V3z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ A.27a)\ (\lambda V4x \in \\
& A.27a.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\
& (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A.27a.((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\
& (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A.27d})^{A.27c}).(\forall V7f \in \\
& ((A.27b^{A.27d})^{A.27c}).(\forall V8z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V8z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod \\
& A.27c\ A.27d))\ (ap\ (c.2Epair.2EUNCURRY\ A.27c\ A.27d\ (ty.2Epair.2Eprod \\
& A.27b\ 2))\ (\lambda V9x \in A.27c.(\lambda V10y \in A.27d.(ap\ (ap\ (c.2Epair.2E.2C \\
& A.27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\
& (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A.27c.(\forall V12y \in A.27d.((p \\
& (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A.27g})^{A.27f})^{A.27e}).(\forall V14f \in (((A.27b^{A.27g})^{A.27f})^{A.27e}). \\
& (\forall V15z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V15z)\ (\\
& ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)))\ (ap\ (c.2Epair.2EUNCURRY\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)\ (ty.2Epair.2Eprod\ A.27b\ 2))\ (\lambda V16w \in A.27e.(ap \\
& (c.2Epair.2EUNCURRY\ A.27f\ A.27g\ (ty.2Epair.2Eprod\ A.27b\ 2)) \\
& (\lambda V17x \in A.27f.(\lambda V18y \in A.27g.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b \\
& 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x) \\
& V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A.27e.(\forall V20x \in \\
& A.27f.(\forall V21y \in A.27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\
& (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty.2Erealax.2Ereal}).((ap\ (c.2Etopology.2Etopspace \\
& ty.2Erealax.2Ereal)\ (ap\ (ap\ (c.2Ereal_topology.2Esubtopology \\
& ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V0s)) = \\
& V0s))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2u \in (2^{ty_2Erealax_2Ereal}). (((p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V1t)) \\
& \quad V0s)) \wedge (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V2u)) V1t))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V2u)) \\
& \quad \quad \quad V0s))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}). ((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V0f) V1s)) \Leftrightarrow (\forall V2t \in (2^{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2Eopen \\
& V2t)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V3x \in ty_2Erealax_2Ereal. \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V3x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V3x)) V2t))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c. \\
& nonempty A.27c \Rightarrow \forall A.27d.nonempty A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty A.27f \Rightarrow \forall A.27g.nonempty A.27g \Rightarrow \\
& \forall A.27h.nonempty A.27h \Rightarrow \forall A.27i.nonempty A.27i \Rightarrow (\\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred_set_2EBIGUNION A.27b) (ap (c.2Epred_set_2EGSPEC \\
& (2^{A.27b}) A.27a) (\lambda V2x \in A.27a.(ap (ap (c.2Epair_2E_2C (2^{A.27b}) \\
& 2) (ap V1f V2x)) (ap V0P V2x)))))) = (ap (c.2Epred_set_2EGSPEC A.27b \\
& A.27b) (\lambda V3a \in A.27b.(ap (ap (c.2Epair_2E_2C A.27b 2) V3a) (\\
& ap (c.2Ebool_2E_3F A.27a) (\lambda V4x \in A.27a.(ap (ap c.2Ebool_2E_2F_5C \\
& (ap V0P V4x)) (ap (ap (c.2Ebool_2EIN A.27b) V3a) (ap V1f V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap (c.2Epred_set_2EBIGUNION A.27e) (ap (c.2Epred_set_2EGSPEC \\
& (2^{A.27e}) (ty_2Epair_2Eprod A.27c A.27d)) (ap (c.2Epair_2EUNCURRY \\
& A.27c A.27d (ty_2Epair_2Eprod (2^{A.27e}) 2)) (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap (ap (c.2Epair_2E_2C (2^{A.27e}) 2) (ap (ap V6f \\
& V7x) V8y)) (ap (ap V5P V7x) V8y)))))) = (ap (c.2Epred_set_2EGSPEC \\
& A.27e A.27e) (\lambda V9a \in A.27e.(ap (ap (c.2Epair_2E_2C A.27e 2) \\
& V9a) (ap (c.2Ebool_2E_3F A.27c) (\lambda V10x \in A.27c.(ap (c.2Ebool_2E_3F \\
& A.27d) (\lambda V11y \in A.27d.(ap (ap c.2Ebool_2E_2F_5C (ap (ap V5P V10x) \\
& V11y)) (ap (ap (c.2Ebool_2EIN A.27e) V9a) (ap (ap V6f V10x) V11y)))))))))) \wedge \\
& (\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}).(\forall V13f \in (((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}). \\
& ((ap (c.2Epred_set_2EBIGUNION A.27i) (ap (c.2Epred_set_2EGSPEC \\
& (2^{A.27i}) (ty_2Epair_2Eprod A.27f (ty_2Epair_2Eprod A.27g A.27h)) \\
& (ap (c.2Epair_2EUNCURRY A.27f (ty_2Epair_2Eprod A.27g A.27h) \\
& (ty_2Epair_2Eprod (2^{A.27i}) 2)) (\lambda V14x \in A.27f.(ap (c.2Epair_2EUNCURRY \\
& A.27g A.27h (ty_2Epair_2Eprod (2^{A.27i}) 2)) (\lambda V15y \in A.27g. \\
& (\lambda V16z \in A.27h.(ap (ap (c.2Epair_2E_2C (2^{A.27i}) 2) (ap (ap \\
& (ap V13f V14x) V15y) V16z)) (ap (ap (ap V12P V14x) V15y) V16z)))))) = \\
& (ap (c.2Epred_set_2EGSPEC A.27i A.27i) (\lambda V17a \in A.27i.(ap \\
& (ap (c.2Epair_2E_2C A.27i 2) V17a) (ap (c.2Ebool_2E_3F A.27f) \\
& (\lambda V18x \in A.27f.(ap (c.2Ebool_2E_3F A.27g) (\lambda V19y \in A.27g. \\
& (ap (c.2Ebool_2E_3F A.27h) (\lambda V20z \in A.27h.(ap (ap c.2Ebool_2E_2F_5C \\
& (ap (ap (ap V12P V18x) V19y) V20z)) (ap (ap (c.2Ebool_2EIN A.27i) \\
& V17a) (ap (ap (ap V13f V18x) V19y) V20z)))))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (70)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_27a).(\forall V1s \in (2^{A_27a}).((p (ap (ap (c_2Etopology_2Eopen_in \\ A_27a) V0top) V1s)) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET A_27a) \\ V1s) (ap (c_2Etopology_2Etopspace A_27a) V0top))))))) \end{aligned} \quad (71)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_27a).(\forall V1k \in (2^{(2^{A_27a})}).(\forall V2s \in (2^{A_27a}). \\ ((p (ap (ap (c_2Ebool_2EIN (2^{A_27a}) V2s) V1k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\ A_27a) V0top) V2s)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in A_27a) \\ V0top) (ap (c_2Epred_set_2EBIGUNION A_27a) V1k)))))) \end{aligned} \quad (72)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{A_27a}). \\ (\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2t \in \\ ((2^{ty_2Erealax_2Ereal})^{A_27a}).(\forall V3s \in (2^{ty_2Erealax_2Ereal}). \\ (\forall V4k \in (2^{A_27a}).((\forall V5i \in A_27a.((p (ap (ap (c_2Ebool_2EIN \\ A_27a) V5i) V4k)) \Rightarrow ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\ (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\ c_2Ereal_topology_2Eeuclidean) V3s)) (ap V2t V5i)))) \wedge (p (ap (\\ ap c_2Ereal_topology_2Econtinuous_on (ap V0f V5i)) (ap V2t V5i)))))) \wedge \\ ((\forall V6i \in A_27a.(\forall V7j \in A_27a.(\forall V8x \in ty_2Erealax_2Ereal. \\ (((p (ap (ap (c_2Ebool_2EIN A_27a) V6i) V4k)) \wedge ((p (ap (ap (c_2Ebool_2EIN \\ A_27a) V7j) V4k)) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ V8x) (ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) (ap (\\ ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) V3s) (ap V2t V6i))) \\ (ap V2t V7j)))))) \Rightarrow ((ap (ap V0f V6i) V8x) = (ap (ap V0f V7j) V8x)))))) \wedge \\ (\forall V9x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ V9x) V3s)) \Rightarrow (\exists V10j \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) \\ V10j) V4k)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V9x) \\ (ap V2t V10j)))) \wedge ((ap V1g V9x) = (ap (ap V0f V10j) V9x)))))) \Rightarrow (p \\ (ap (ap c_2Ereal_topology_2Econtinuous_on V1g) V3s)))))) \end{aligned}$$