

thm_2Ereal__topology_2EPASTING__LEMMA__CLOSED (TMY2w6VT6VksRhemSde64M1N7BAyMhqonpZ)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a}))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.\lambda 27c^{A.\lambda 27b})^{A.\lambda 27a}))$

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.(\lambda V0x \in A.\lambda 27a.(\lambda V1y \in A.\lambda 27b.V0x))$

Definition 6 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda 27a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda 27a (A.\lambda 27a^{A.\lambda 27a})) A.\lambda 27a))$

Definition 7 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.\lambda 27a}).(ap (ap (c_2Emin_2E_3D (2^{A.\lambda 27a})) (\lambda V1f \in (2^{A.\lambda 27a}).(ap V1f V0x))$

Definition 9 We define $c_2Ecombin_2Eo$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in (A.\lambda 27b^{A.\lambda 27c}).\lambda V1g \in (A.\lambda 27c^{A.\lambda 27b}))$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.\lambda 27a.(\lambda V1f \in (2^{A.\lambda 27a}).(ap V1f V0x))$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c_2Epair_2EABS_prod A.\lambda 27a A.\lambda 27b \in ((ty_2Epair_2Eprod A.\lambda 27a A.\lambda 27b)^{(2^{A.\lambda 27b})^{A.\lambda 27a}}) \tag{2}$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(3)

Definition 14 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

Definition 15 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal$$
(4)

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal})$$
(7)

Definition 16 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap (c_2Emin_2E_40 : \iota \Rightarrow \iota)$

Let $c_2Erealx_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(8)

Definition 17 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(11)

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 20 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_2Ebool_2E_2$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (14)$$

Definition 21 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax_2Ereal$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in ((2^{(2^{A_27a})})^{(ty_2Etopology_2Etopology A_27a)}) \quad (15)$$

Definition 22 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 23 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2ET)$.

Definition 24 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 25 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_2E3D_2E3D_2E3E V0t) c_2Ebool_2E_2E7E$

Definition 26 We define $c_2Epred_set_2E_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Definition 27 We define $c_2Ereal_topology_2E_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap c_2Ereal_topology$

Definition 28 We define $c_2Ereal_topology_2E_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Ereal$

Let $c_2Epair_2E_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2E_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (16)$$

Let $c_2Epair_2E_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2E_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (17)$$

Definition 29 We define $c_2Epair_2E_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a$

Definition 30 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 31 We define $c_Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c_Epred_set_2E$

Definition 32 We define $c_Etopology_2Etopspace$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology_2E$

Definition 33 We define $c_Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_E$

Definition 34 We define $c_Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology_2E$

Definition 35 We define $c_Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A-27a}).(ap (c_E$

Definition 36 We define $c_Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF)$.

Definition 37 We define $c_Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_Ebool_2E_21 2)$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \tag{21}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in \\
& A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).(((p \ V0P) \wedge (\forall V2x \in A_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\
& A_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).(((p \ V0P) \vee (\exists V2x \in A_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in \\
& A_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))
\end{aligned} \tag{34}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\exists V2y \in \\ A_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}).(\\ \forall V4x \in A_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2EI \\ A_27a)\ V0x) = V0x)) \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\ A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\ A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\ A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}).(\forall V1v \in \\ A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap\ (ap\ (c_2Epair_2E_2C \\ A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ (2^{A_27a}).(\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0P \in (2^{A.27a}). (\forall V1f \in (A.27a^{A.27b}). (\forall V2s \in \\
& \quad (2^{A.27b}). (\forall V3y \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\
& \quad V3y)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27b\ A.27a)\ V1f)\ V2s))) \Rightarrow (\\
& \quad p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A.27b. ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad A.27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a) \\
& \quad V0s)) \Rightarrow (\forall V1f \in (A.27b^{A.27a}). (p\ (ap\ (c.2Epred_set.2EFINITE \\
& \quad A.27b)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty.2Erealax.2Ereal}). ((ap\ (c.2Etopology.2Etopspace \\
& \quad ty.2Erealax.2Ereal)\ (ap\ (ap\ (c.2Ereal_topology.2Esubtopology \\
& \quad ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V0s)) = \\
& \quad V0s))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty.2Erealax.2Ereal}). (\forall V1t \in (2^{ty.2Erealax.2Ereal}). \\
& \quad (\forall V2u \in (2^{ty.2Erealax.2Ereal}). (((p\ (ap\ (ap\ (c.2Etopology.2Eclosed_in \\
& \quad ty.2Erealax.2Ereal)\ (ap\ (ap\ (c.2Ereal_topology.2Esubtopology \\
& \quad ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V1t)) \\
& \quad V0s)) \wedge (p\ (ap\ (ap\ (c.2Etopology.2Eclosed_in\ ty.2Erealax.2Ereal) \\
& \quad (ap\ (ap\ (c.2Ereal_topology.2Esubtopology\ ty.2Erealax.2Ereal) \\
& \quad c.2Ereal_topology.2Eeuclidean)\ V2u))\ V1t)))) \Rightarrow (p\ (ap\ (ap\ (c.2Etopology.2Eclosed_in \\
& \quad ty.2Erealax.2Ereal)\ (ap\ (ap\ (c.2Ereal_topology.2Esubtopology \\
& \quad ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V2u)) \\
& \quad V0s))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}). (\forall V1s \in \\
& \quad (2^{ty.2Erealax.2Ereal}). ((p\ (ap\ (ap\ c.2Ereal_topology.2Econtinuous_on \\
& \quad V0f)\ V1s)) \Leftrightarrow (\forall V2t \in (2^{ty.2Erealax.2Ereal}). ((p\ (ap\ c.2Ereal_topology.2Eclosed \\
& \quad V2t)) \Rightarrow (p\ (ap\ (ap\ (c.2Etopology.2Eclosed_in\ ty.2Erealax.2Ereal) \\
& \quad (ap\ (ap\ (c.2Ereal_topology.2Esubtopology\ ty.2Erealax.2Ereal) \\
& \quad c.2Ereal_topology.2Eeuclidean)\ V1s))\ (ap\ (c.2Epred_set.2EGSPEC \\
& \quad ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ (\lambda V3x \in ty.2Erealax.2Ereal. \\
& \quad (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ 2)\ V3x)\ (ap\ (ap\ c.2Ebool.2E.2F.5C \\
& \quad (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealax.2Ereal)\ V3x)\ V1s))\ (ap\ (ap\ (\\
& \quad c.2Ebool.2EIN\ ty.2Erealax.2Ereal)\ (ap\ V0f\ V3x))\ V2t)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow (\\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred_set_2EBIGUNION\ A.27b)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27b})\ A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c.2Epred_set_2EGSPEC\ A.27b \\
& A.27b)\ (\lambda V3a \in A.27b.(ap\ (ap\ (c.2Epair_2E_2C\ A.27b\ 2)\ V3a)\ (\\
& ap\ (c.2Ebool_2E_3F\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27e)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27e})\ (ty_2Epair_2Eprod\ A.27c\ A.27d))\ (ap\ (c.2Epair_2EUNCURRY \\
& A.27c\ A.27d\ (ty_2Epair_2Eprod\ (2^{A.27e})\ 2))\ (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c.2Epred_set_2EGSPEC \\
& A.27e\ A.27e)\ (\lambda V9a \in A.27e.(ap\ (ap\ (c.2Epair_2E_2C\ A.27e\ 2) \\
& V9a)\ (ap\ (c.2Ebool_2E_3F\ A.27c)\ (\lambda V10x \in A.27c.(ap\ (c.2Ebool_2E_3F \\
& A.27d)\ (\lambda V11y \in A.27d.(ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ V5P\ V10x) \\
& V11y))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27e)\ V9a)\ (ap\ (ap\ V6f\ V10x)\ V11y)))))))))) \wedge \\
& ((\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}).(\forall V13f \in (((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27i)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27i})\ (ty_2Epair_2Eprod\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h)) \\
& (ap\ (c.2Epair_2EUNCURRY\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h) \\
& (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V14x \in A.27f.(ap\ (c.2Epair_2EUNCURRY \\
& A.27g\ A.27h\ (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V15y \in A.27g. \\
& (\lambda V16z \in A.27h.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27i})\ 2)\ (ap\ (ap \\
& (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x)\ V15y)\ V16z)))))) = \\
& (ap\ (c.2Epred_set_2EGSPEC\ A.27i\ A.27i)\ (\lambda V17a \in A.27i.(ap \\
& (ap\ (c.2Epair_2E_2C\ A.27i\ 2)\ V17a)\ (ap\ (c.2Ebool_2E_3F\ A.27f) \\
& (\lambda V18x \in A.27f.(ap\ (c.2Ebool_2E_3F\ A.27g)\ (\lambda V19y \in A.27g. \\
& (ap\ (c.2Ebool_2E_3F\ A.27h)\ (\lambda V20z \in A.27h.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (71)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0top \in (ty.2Etopology.2Etopology \\ A.27a).(\forall V1s \in (2^{A.27a}).((p (ap (ap (c.2Etopology.2Eclosed_in \\ A.27a) V0top) V1s)) \Rightarrow (p (ap (ap (c.2Epred_set.2ESUBSET \ A.27a) \\ V1s) (ap (c.2Etopology.2Etopspace \ A.27a) V0top))))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0top \in (ty.2Etopology.2Etopology \\ A.27a).(\forall V1s \in (2^{(2^{A.27a})}).(((p (ap (c.2Epred_set.2EFINITE \\ (2^{A.27a}) V1s)) \wedge (\forall V2t \in (2^{A.27a}).((p (ap (ap (c.2Ebool.2EIN \\ (2^{A.27a}) V2t) V1s)) \Rightarrow (p (ap (ap (c.2Etopology.2Eclosed_in \ A.27a) \\ V0top) V2t)))))) \Rightarrow (p (ap (ap (c.2Etopology.2Eclosed_in \ A.27a) \\ V0top) (ap (c.2Epred_set.2EBIGUNION \ A.27a) V1s))))))) \end{aligned} \quad (73)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0f \in ((ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal})^{A.27a}). \\ (\forall V1g \in (ty.2Erealax.2Ereal^{ty.2Erealax.2Ereal}).(\forall V2t \in \\ ((2^{ty.2Erealax.2Ereal})^{A.27a}).(\forall V3s \in (2^{ty.2Erealax.2Ereal}). \\ (\forall V4k \in (2^{A.27a}).(((p (ap (c.2Epred_set.2EFINITE \ A.27a) \\ V4k)) \wedge ((\forall V5i \in A.27a.((p (ap (ap (c.2Ebool.2EIN \ A.27a) V5i) \\ V4k)) \Rightarrow ((p (ap (ap (c.2Etopology.2Eclosed_in \ ty.2Erealax.2Ereal) \\ (ap (ap (c.2Ereal_topology.2Esubtopology \ ty.2Erealax.2Ereal) \\ c.2Ereal_topology.2Euclidean) V3s)) (ap V2t V5i)))) \wedge (p (ap (\\ ap \ c.2Ereal_topology.2Econtinuous_on (ap V0f V5i)) (ap V2t V5i)))))) \wedge \\ ((\forall V6i \in A.27a.(\forall V7j \in A.27a.(\forall V8x \in ty.2Erealax.2Ereal. \\ (((p (ap (ap (c.2Ebool.2EIN \ A.27a) V6i) V4k)) \wedge ((p (ap (ap (c.2Ebool.2EIN \\ A.27a) V7j) V4k)) \wedge (p (ap (ap (c.2Ebool.2EIN \ ty.2Erealax.2Ereal) \\ V8x) (ap (ap (c.2Epred_set.2EINTER \ ty.2Erealax.2Ereal) (ap (\\ ap (c.2Epred_set.2EINTER \ ty.2Erealax.2Ereal) V3s) (ap V2t V6i))) \\ (ap V2t V7j)))))) \Rightarrow ((ap (ap V0f V6i) V8x) = (ap (ap V0f V7j) V8x)))))) \wedge \\ (\forall V9x \in ty.2Erealax.2Ereal.((p (ap (ap (c.2Ebool.2EIN \ ty.2Erealax.2Ereal) \\ V9x) V3s)) \Rightarrow (\exists V10j \in A.27a.((p (ap (ap (c.2Ebool.2EIN \ A.27a) \\ V10j) V4k)) \wedge ((p (ap (ap (c.2Ebool.2EIN \ ty.2Erealax.2Ereal) V9x) \\ (ap V2t V10j)))) \wedge ((ap V1g V9x) = (ap (ap V0f V10j) V9x)))))) \Rightarrow (\\ p (ap (ap \ c.2Ereal_topology.2Econtinuous_on V1g) V3s)))))) \end{aligned}$$