

thm\_2Ereal\_\_topology\_2EPROPER\_\_MAP  
(TMPpkcPn-  
FSw6PMkiHVy1hWbwBLiacu7CpPA)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let `c_2Earithmetic_2EEVEN` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let `c_2Earithmetic_2EODD` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}))) P))$

**Definition 5** We define `c_2Ebool_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2. V0t))$ .

**Definition 6** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

**Definition 8** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (6)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 16** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 17** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 18** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 19** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (11)$$

**Definition 20** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 21** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 22** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 23** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (12)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (13)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (14)$$

**Definition 24** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})^{A\_27b})$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (15)$$

**Definition 25** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (16)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealax\_2Ereal \quad (17)$$

**Definition 26** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (18)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (19)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (20)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ t))$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)}) \quad (21)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (22)$$

**Definition 29** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap\ (c\_2Ebool\_2E2))$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (23)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A\_27a)}})) \quad (24)$$

**Definition 30** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax))$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in ((2^{(2^A\_27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (25)$$

**Definition 31** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E2))$

**Definition 32** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology\ A\_27a)$

**Definition 33** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2EF)$ .

**Definition 34** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E2EF))$

**Definition 35** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 36** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap (a$

**Definition 37** We define  $c\_2Ereal\_topology\_2Elimit\_point\_of$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1s \in ($

**Definition 38** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_s$

**Definition 39** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 40** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ereal\_topology\_2Enet A0) \quad (26)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ A\_27a \in ((ty\_2Ereal\_topology\_2Enet A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (27)$$

**Definition 41** We define  $c\_2Ereal\_topology\_2Esequentially$  to be  $(ap (c\_2Ereal\_topology\_2Emk\_net ty\_2E$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ereal\_topology\_2Enetord A\_27a \in ((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_topology\_2Enet A\_27a)} \quad (28)$$

**Definition 42** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology$

**Definition 43** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2E$

**Definition 44** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A$

**Definition 45** We define  $c\_2Ereal\_topology\_2EClosed$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap c\_2Ereal\_topo$

**Definition 46** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_s$

**Definition 47** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 (2$

**Definition 48** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1,$

**Definition 49** We define  $c\_2Ereal\_topology\_2Ecompact$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E$

**Definition 50** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 51** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap ($

**Definition 52** We define  $c\_2Etopology\_2EClosed\_in$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
(ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))))) \\
& \tag{29}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
V1n) V0m)))) \\
& \tag{30}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
V1n) V0m)))) \\
& \tag{31}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
\forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
(ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
& \tag{32}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V0n))) \\
& \tag{33}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
(((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
(((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
(((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
(ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
(ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
V0m) V1n))))))))) \\
& \tag{34}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& \quad \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \quad (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
& \quad \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& \quad V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& \quad \quad V1n)) V0m)))))) \quad (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
& \quad c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad \quad c\_2Earithmetic\_2EZERO))) V0n))) \quad (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\
& \quad (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad \quad V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \quad (40)
\end{aligned}$$

Assume the following.

$$\text{True} \quad (41)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (42)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (45)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (50)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (51)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (52)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (53)$$



Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (54)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (56)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\forall V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (57)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\exists V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (58)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a. (p\ (ap\ V1Q\ V4x)))))) \quad (59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. (((\forall V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (60)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). (((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (61)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. ((\exists V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (62)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x)) \vee (p V0Q)))))) \quad (63)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (70)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (71)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (72)$$

Assume the following.

$$(\forall V0y \in 2.(\forall V1x \in 2.(((p V0y) \Rightarrow (p V1x)) \Leftrightarrow ((\neg(p V1x)) \Rightarrow (\neg(p V0y)))))) \quad (73)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (74)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (75)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (76)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p V0P) \Rightarrow (\exists V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p V0P) \Rightarrow (p (ap V1Q V3x))))))) \quad (77)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1x \in A\_27a.((p (ap (ap (c\_2Epred\_set\_2ESUBSET A\_27a) (ap (ap (c\_2Epred\_set\_2EINSERT A\_27a) V1x) (c\_2Epred\_set\_2EEMPTY A\_27a))) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1x) V0s)))))) \quad (78)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m)) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\
& A\_27b. (((ap (ap (c\_2Epair\_2E\_2C A\_27a A\_27b) V0x) V1y) = (ap (ap \\
& (c\_2Epair\_2E\_2C A\_27a A\_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN \\
& A\_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) V1t))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0f \in ((ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}). (\forall V1v \in \\
& A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1v) (ap (c\_2Epred\_set\_2EGSPEC \\
& A\_27a A\_27b) V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap (ap (c\_2Epair\_2E\_2C \\
& A\_27a 2) V1v) c\_2Ebool\_2ET) = (ap V0f V2x))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg (p (ap (ap \\
& (c\_2Ebool\_2EIN A\_27a) V0x) (c\_2Epred\_set\_2EEMPTY A\_27a))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). ((\exists V1x \in \\
& A\_27a. (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1x) V0s))) \Leftrightarrow (\neg (V0s = (c\_2Epred\_set\_2EEMPTY \\
& A\_27a))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred\_set.2EUNIV\ A.27a)))) \quad (86)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ \\ V2x)\ (ap\ (ap\ (c.2Epred\_set.2EINTER\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ ( \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ \\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ \\ V2x)\ (ap\ (ap\ (c.2Epred\_set.2EDIFF\ A.27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ ( \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN\ \\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ A.27a.(\forall V2s \in (2^{A.27a}).((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ \\ V0x)\ (ap\ (ap\ (c.2Epred\_set.2EINSERT\ A.27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\ V1y) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V2s)))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1x \in \\ A.27a.(\forall V2y \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ \\ (ap\ (ap\ (c.2Epred\_set.2EDELETE\ A.27a)\ V0s)\ V2y)))) \Leftrightarrow ((p\ (ap\ (ap\ \\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0y \in A.27b.(\forall V1s \in (2^{A.27a}).(\forall V2f \in (A.27b^{A.27a}). \\ ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c.2Epred\_set.2EIMAGE\ \\ A.27a\ A.27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V3x)\ V1s)))))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in (2^{A.27a}).(\forall V1f \in (A.27a^{A.27b}).(\forall V2s \in \\ (2^{A.27b}).(\forall V3y \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ \\ V3y)\ (ap\ (ap\ (c.2Epred\_set.2EIMAGE\ A.27b\ A.27a)\ V1f)\ V2s)))) \Rightarrow ( \\ p\ (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ \\ A.27b)\ V4x)\ V2s)) \Rightarrow (p\ (ap\ V0P\ (ap\ V1f\ V4x)))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0s \in (2^{A-27a}).((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a) \\ & \quad V0s)) \Rightarrow (\forall V1f \in (A\_27b^{A-27a}).(p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27b)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27b)\ V1f)\ V0s)))))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1sos \in \\ & \quad (2^{(2^{A-27a})}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Epred\_set\_2EBIGUNION \\ & \quad A\_27a)\ V1sos)))) \Leftrightarrow (\exists V2s \in (2^{A-27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A-27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (94)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).(( \\ & \quad p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (ap\ (c\_2Epred\_set\_2EBIGUNION \\ & \quad A\_27a)\ V0P)))) \Leftrightarrow ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ (2^{A-27a})\ V0P)) \wedge \\ & \quad (\forall V1s \in (2^{A-27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ (2^{A-27a}) \\ & \quad V1s)\ V0P)) \Rightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V1s)))))) \end{aligned} \quad (95)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0P \in (2^{(2^{A-27a})}).(\forall V1f \in (A\_27a^{A-27b}).(\forall V2s \in \\ & \quad (2^{A-27b}).((\forall V3t \in (2^{A-27a}).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27a)\ V3t)) \wedge (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V3t)\ (ap \\ & \quad (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27b\ A\_27a)\ V1f)\ V2s)))))) \Rightarrow (p\ (ap\ V0P \\ & \quad V3t)))) \Leftrightarrow (\forall V4t \in (2^{A-27b}).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27b)\ V4t)) \wedge (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27b)\ V4t)\ V2s))) \Rightarrow \\ & \quad (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27b\ A\_27a)\ V1f)\ V4t)))))) \end{aligned} \quad (96)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty \\
& A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow \\
& (\forall V0Q \in (2^{A\_27b}). (\forall V1P \in (2^{A\_27a}). (\forall V2f \in \\
& (A\_27b^{A\_27a}). (\forall V3z \in A\_27b. ((p (ap (ap (c\_2Ebool\_2EIN \\
& A\_27b) V3z) (ap (c\_2Epred\_set\_2EGSPEC\ A\_27b\ A\_27a) (\lambda V4x \in \\
& A\_27a. (ap (ap (c\_2Epair\_2E\_2C\ A\_27b\ 2) (ap V2f\ V4x)) (ap V1P\ V4x)))))) \Rightarrow \\
& (p (ap V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A\_27a. ((p (ap V1P\ V5x)) \Rightarrow (p (ap V0Q \\
& (ap V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A\_27d})^{A\_27c}). (\forall V7f \in \\
& ((A\_27b^{A\_27d})^{A\_27c}). (\forall V8z \in A\_27b. ((p (ap (ap (c\_2Ebool\_2EIN \\
& A\_27b) V8z) (ap (c\_2Epred\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod \\
& A\_27c\ A\_27d)) (ap (c\_2Epair\_2EUNCURRY\ A\_27c\ A\_27d\ (ty\_2Epair\_2Eprod \\
& A\_27b\ 2)) (\lambda V9x \in A\_27c. (\lambda V10y \in A\_27d. (ap (ap (c\_2Epair\_2E\_2C \\
& A\_27b\ 2) (ap (ap V7f\ V9x) V10y)) (ap (ap V6P\ V9x) V10y)))))) \Rightarrow (p \\
& (ap V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A\_27c. (\forall V12y \in A\_27d. ((p \\
& (ap (ap V6P\ V11x) V12y)) \Rightarrow (p (ap V0Q (ap (ap V7f\ V11x) V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A\_27g})^{A\_27f})^{A\_27e}). (\forall V14f \in (((A\_27b^{A\_27g})^{A\_27f})^{A\_27e}). \\
& (\forall V15z \in A\_27b. ((p (ap (ap (c\_2Ebool\_2EIN\ A\_27b) V15z) ( \\
& ap (c\_2Epred\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27e\ (ty\_2Epair\_2Eprod \\
& A\_27f\ A\_27g))) (ap (c\_2Epair\_2EUNCURRY\ A\_27e\ (ty\_2Epair\_2Eprod \\
& A\_27f\ A\_27g) (ty\_2Epair\_2Eprod\ A\_27b\ 2)) (\lambda V16w \in A\_27e. (ap \\
& (c\_2Epair\_2EUNCURRY\ A\_27f\ A\_27g\ (ty\_2Epair\_2Eprod\ A\_27b\ 2)) \\
& (\lambda V17x \in A\_27f. (\lambda V18y \in A\_27g. (ap (ap (c\_2Epair\_2E\_2C\ A\_27b \\
& 2) (ap (ap (ap V14f\ V16w) V17x) V18y)) (ap (ap (ap V13P\ V16w) V17x) \\
& V18y)))))) \Rightarrow (p (ap V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A\_27e. (\forall V20x \in \\
& A\_27f. (\forall V21y \in A\_27g. ((p (ap (ap (ap V13P\ V19w) V20x) V21y)) \Rightarrow \\
& (p (ap V0Q (ap (ap (ap V14f\ V19w) V20x) V21y)))))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}). \\
& (\forall V1s \in (2^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE\ A\_27a) \\
& V1s)) \Rightarrow (\exists V2a \in ty\_2Enum\_2Enum. (\forall V3x \in A\_27a. ((p ( \\
& ap (ap (c\_2Ebool\_2EIN\ A\_27a) V3x) V1s)) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap V0f\ V3x)) V2a))))))
\end{aligned} \tag{98}$$



Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow ( \\
& \quad \forall V0f \in ((2^{A_{.27b}})^{A_{.27a}}).(\forall V1s \in (2^{A_{.27a}}).((ap\ ( \\
& \quad c_{.2}Epred\_set\_2EBIGINTER\ A_{.27b})\ (ap\ (ap\ (c_{.2}Epred\_set\_2EIMAGE \\
& \quad A_{.27a}\ (2^{A_{.27b}}))\ V0f)\ V1s)) = (ap\ (c_{.2}Epred\_set\_2EGSPEC\ A_{.27b} \\
& \quad A_{.27b})\ (\lambda V2y \in A_{.27b}.(ap\ (ap\ (c_{.2}Epair\_2E\_2C\ A_{.27b}\ 2)\ V2y)\ ( \\
& \quad ap\ (c_{.2}Ebool\_2E\_21\ A_{.27a})\ (\lambda V3x \in A_{.27a}.(ap\ (ap\ c_{.2}Emin\_2E\_3D\_3D\_3E \\
& \quad (ap\ (ap\ (c_{.2}Ebool\_2EIN\ A_{.27a})\ V3x)\ V1s))\ (ap\ (ap\ (c_{.2}Ebool\_2EIN \\
& \quad A_{.27b})\ V2y)\ (ap\ V0f\ V3x)))))))))) \\
& \hspace{15em} (99)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\
& \quad A_{.27a}).(\forall V1s \in (2^{A_{.27a}}).(\forall V2t \in (2^{A_{.27a}}).((p\ ( \\
& \quad ap\ (ap\ (c_{.2}Etopology\_2Eclosed\_in\ A_{.27a})\ (ap\ (ap\ (c_{.2}Ereal\_topology\_2Esubtopology \\
& \quad A_{.27a})\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_{.2}Epred\_set\_2ESUBSET\ A_{.27a}) \\
& \quad V2t)\ V1s)))))) \\
& \hspace{15em} (100)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (2^{(2^{ty\_2Erealax\_2Ereal})}).((\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p\ (ap\ (ap\ (c_{.2}Ebool\_2EIN\ (2^{ty\_2Erealax\_2Ereal}))\ V1s)\ V0f)) \Rightarrow \\
& \quad (p\ (ap\ c_{.2}Ereal\_topology\_2EOpen\ V1s)))) \Rightarrow (p\ (ap\ c_{.2}Ereal\_topology\_2EOpen \\
& \quad (ap\ (c_{.2}Epred\_set\_2EBIGUNION\ ty\_2Erealax\_2Ereal)\ V0f)))) \\
& \hspace{15em} (101)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((ap\ (c_{.2}Etopology\_2Etopspace \\
& \quad ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (c_{.2}Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal)\ c_{.2}Ereal\_topology\_2Euclidean)\ V0s)) = \\
& \quad V0s)) \\
& \hspace{15em} (102)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow ( \\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred\_set.2EBIGINTER\ A.27b)\ (ap\ (c.2Epred\_set.2EGSPEC \\
& (2^{A.27b})\ A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ (c.2Epair\_2E\_2C\ (2^{A.27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c.2Epred\_set.2EGSPEC\ A.27b \\
& A.27b)\ (\lambda V3a \in A.27b.(ap\ (ap\ (c.2Epair\_2E\_2C\ A.27b\ 2)\ V3a)\ ( \\
& ap\ (c.2Ebool\_2E\_21\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Emin\_2E\_3D\_3D\_3E \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap\ (c.2Epred\_set.2EBIGINTER\ A.27e)\ (ap\ (c.2Epred\_set.2EGSPEC \\
& (2^{A.27e})\ (ty.2Epair\_2Eprod\ A.27c\ A.27d))\ (ap\ (c.2Epair\_2EUNCURRY \\
& A.27c\ A.27d\ (ty.2Epair\_2Eprod\ (2^{A.27e})\ 2))\ (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap\ (ap\ (c.2Epair\_2E\_2C\ (2^{A.27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c.2Epred\_set.2EGSPEC \\
& A.27e\ A.27e)\ (\lambda V9a \in A.27e.(ap\ (ap\ (c.2Epair\_2E\_2C\ A.27e\ 2)\ \\
& V9a)\ (ap\ (c.2Ebool\_2E\_21\ A.27c)\ (\lambda V10x \in A.27c.(ap\ (c.2Ebool\_2E\_21 \\
& A.27d)\ (\lambda V11y \in A.27d.(ap\ (ap\ c.2Emin\_2E\_3D\_3D\_3E\ (ap\ (ap\ V5P \\
& V10x)\ V11y))\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27e)\ V9a)\ (ap\ (ap\ V6f\ V10x) \\
& V11y)))))))))) \wedge (\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}). \\
& (\forall V13f \in (((((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}).((ap\ (c.2Epred\_set.2EBIGINTER \\
& A.27i)\ (ap\ (c.2Epred\_set.2EGSPEC\ (2^{A.27i})\ (ty.2Epair\_2Eprod \\
& A.27f\ (ty.2Epair\_2Eprod\ A.27g\ A.27h))\ (ap\ (c.2Epair\_2EUNCURRY \\
& A.27f\ (ty.2Epair\_2Eprod\ A.27g\ A.27h)\ (ty.2Epair\_2Eprod\ (2^{A.27i}) \\
& 2))\ (\lambda V14x \in A.27f.(ap\ (c.2Epair\_2EUNCURRY\ A.27g\ A.27h\ (ty.2Epair\_2Eprod \\
& (2^{A.27i})\ 2))\ (\lambda V15y \in A.27g.(\lambda V16z \in A.27h.(ap\ (ap\ (c.2Epair\_2E\_2C \\
& (2^{A.27i})\ 2)\ (ap\ (ap\ (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x) \\
& V15y)\ V16z)))))))))) = (ap\ (c.2Epred\_set.2EGSPEC\ A.27i\ A.27i) \\
& (\lambda V17a \in A.27i.(ap\ (ap\ (c.2Epair\_2E\_2C\ A.27i\ 2)\ V17a)\ (ap\ (c.2Ebool\_2E\_21 \\
& A.27f)\ (\lambda V18x \in A.27f.(ap\ (c.2Ebool\_2E\_21\ A.27g)\ (\lambda V19y \in \\
& A.27g.(ap\ (c.2Ebool\_2E\_21\ A.27h)\ (\lambda V20z \in A.27h.(ap\ (ap\ c.2Emin\_2E\_3D\_3D\_3E \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty.2Erealax.2Ereal}).(\forall V1u \in (2^{ty.2Erealax.2Ereal}). \\
& ((p\ (ap\ (ap\ (c.2Etopology.2Eopen\_in\ ty.2Erealax.2Ereal)\ (ap \\
& (ap\ (c.2Ereal\_topology.2Esubtopology\ ty.2Erealax.2Ereal) \\
& c.2Ereal\_topology.2Euclidean)\ V1u))\ V0s)) \Leftrightarrow (\exists V2t \in ( \\
& 2^{ty.2Erealax.2Ereal}).((p\ (ap\ c.2Ereal\_topology.2Eopen\ V2t)) \wedge \\
& (V0s = (ap\ (ap\ (c.2Epred\_set.2EINTER\ ty.2Erealax.2Ereal)\ V1u) \\
& V2t))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0u \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (c\_2Ereal\_topology\_2EOpen V1s)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in \\
& ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V0u)) \\
& (ap (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) V0u) V1s))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1u \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) ( \\
& ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) V1u)) V0s)) \Leftrightarrow (\exists V2t \in ( \\
& 2^{ty\_2Erealax\_2Ereal}).((p (ap (c\_2Ereal\_topology\_2EClosed \\
& V2t)) \wedge (V0s = (ap (ap (c\_2Epred\_set\_2EINTER ty\_2Erealax\_2Ereal) \\
& V1u) V2t))))))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) ( \\
& ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) V1t)) V0s)) \Leftrightarrow ((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& ty\_2Erealax\_2Ereal) V0s) V1t)) \wedge (\forall V2x \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap (c\_2Ereal\_topology\_2Elimit\_point\_of V2x) V0s)) \wedge \\
& (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V1t)))) \Rightarrow (p ( \\
& ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V0s))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Ereal\_topology\_2Elimit\_point\_of V0x) V1s)) \Leftrightarrow \\
& (\exists V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}).(\forall V3n \in \\
& ty\_2Eenum\_2Eenum.(p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& (ap V2f V3n)) (ap (ap (c\_2Epred\_set\_2EDELETE ty\_2Erealax\_2Ereal) \\
& V1s) V0x)))) \wedge ((\forall V4m \in ty\_2Eenum\_2Eenum.(\forall V5n \in ty\_2Eenum\_2Eenum. \\
& (((ap V2f V4m) = (ap V2f V5n)) \Leftrightarrow (V4m = V5n)))) \wedge (p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& ty\_2Eenum\_2Eenum) V2f) V0x) c\_2Ereal\_topology\_2Esequentially))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.(\forall V2k \in ty\_2Eenum\_2Eenum.((p (ap (ap \\
& (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E ty\_2Eenum\_2Eenum) V0f) V1l) \\
& c\_2Ereal\_topology\_2Esequentially)) \Rightarrow (p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& ty\_2Eenum\_2Eenum) (\lambda V3i \in ty\_2Eenum\_2Eenum.(ap V0f (ap (ap (c\_2Earithmetic\_2E\_2B \\
& V3i) V2k)))) V1l) c\_2Ereal\_topology\_2Esequentially))))))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap c\_2Ereal\_topology\_2Ecompact \\
& \quad V0s)) \Leftrightarrow (\forall V1f \in (2^{(2^{ty\_2Erealax\_2Ereal})}).((\forall V2t \in \\
& (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Ebool\_2EIN (2^{ty\_2Erealax\_2Ereal}) \\
& \quad V2t) V1f)) \Rightarrow (p (ap c\_2Ereal\_topology\_2EOpen V2t)))) \wedge (p (ap (ap \\
& (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V0s) (ap (c\_2Epred\_set\_2EBIGUNION \\
& \quad ty\_2Erealax\_2Ereal) V1f)))) \Rightarrow (\exists V3f\_27 \in (2^{(2^{ty\_2Erealax\_2Ereal})}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET (2^{ty\_2Erealax\_2Ereal}) \\
& \quad V3f\_27) V1f)) \wedge (p (ap (c\_2Epred\_set\_2EFINITE (2^{ty\_2Erealax\_2Ereal}) \\
& \quad V3f\_27)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& \quad V0s) (ap (c\_2Epred\_set\_2EBIGUNION ty\_2Erealax\_2Ereal) V3f\_27)))))))))) \\
& \hspace{15em} (110)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad ty\_2Enum\_2Enum) V0f) V1l) c\_2Ereal\_topology\_2Esequentially)) \Rightarrow \\
& (p (ap c\_2Ereal\_topology\_2Ecompact (ap (ap (c\_2Epred\_set\_2EINSERT \\
& \quad ty\_2Erealax\_2Ereal) V1l) (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Enum\_2Enum \\
& \quad ty\_2Erealax\_2Ereal) V0f) (c\_2Epred\_set\_2EUNIV ty\_2Enum\_2Enum))))))))) \\
& \hspace{15em} (111)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& (((p (ap c\_2Ereal\_topology\_2Eclosed V0s)) \wedge (p (ap c\_2Ereal\_topology\_2Ecompact \\
& \quad V1t))) \Rightarrow (p (ap c\_2Ereal\_topology\_2Ecompact (ap (ap (c\_2Epred\_set\_2EINTER \\
& \quad ty\_2Erealax\_2Ereal) V0s) V1t)))))) \\
& \hspace{15em} (112)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal.(p (ap c\_2Ereal\_topology\_2Ecompact \\
& (ap (ap (c\_2Epred\_set\_2EINSERT ty\_2Erealax\_2Ereal) V0a) (c\_2Epred\_set\_2EEMPTY \\
& \quad ty\_2Erealax\_2Ereal)))))) \\
& \hspace{15em} (113)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (2^{(2^{ty\_2Erealax\_2Ereal})}).((\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Ebool\_2EIN (2^{ty\_2Erealax\_2Ereal}) V1t) V0f)) \Rightarrow \\
& \quad (p (ap c\_2Ereal\_topology\_2Ecompact V1t)))) \wedge (\forall V2f\_27 \in \\
& \quad (2^{(2^{ty\_2Erealax\_2Ereal})}).((p (ap (c\_2Epred\_set\_2EFINITE \\
& \quad (2^{ty\_2Erealax\_2Ereal}) V2f\_27)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad (2^{ty\_2Erealax\_2Ereal}) V2f\_27) V0f))) \Rightarrow (\neg((ap (c\_2Epred\_set\_2EBIGINTER \\
& \quad ty\_2Erealax\_2Ereal) V2f\_27) = (c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal)))))) \Rightarrow \\
& \quad (\neg((ap (c\_2Epred\_set\_2EBIGINTER ty\_2Erealax\_2Ereal) V0f) = \\
& \quad (c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal)))))) \\
& \hspace{15em} (114)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (115)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (116)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (117)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (118)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (119)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (120)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (121)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (122)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (123)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (124)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (125)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (126)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (127)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (128)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (129)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}), (\forall V1s \in \\ & \quad (2^{ty\_2Erealax\_2Ereal}), (\forall V2t \in (2^{ty\_2Erealax\_2Ereal}), \\ & \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap ( \\ & \quad ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & \quad V0f) V1s)) V2t)) \Rightarrow ((\forall V3k \in (2^{ty\_2Erealax\_2Ereal}), ((p \\ & \quad (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V3k) V2t)) \wedge \\ & \quad (p (ap c\_2Ereal\_topology\_2Ecompact V3k))) \Rightarrow (p (ap c\_2Ereal\_topology\_2Ecompact \\ & \quad (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & \quad (\lambda V4x \in ty\_2Erealax\_2Ereal. (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & \quad 2) V4x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\ & \quad V4x) V1s)) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V4x)) \\ & \quad V3k)))))) \Leftrightarrow ((\forall V5k \in (2^{ty\_2Erealax\_2Ereal}), ((p (ap \\ & \quad (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap (ap ( \\ & \quad c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) \\ & \quad V1s)) V5k)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\ & \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\ & \quad c\_2Ereal\_topology\_2Eeuclidean) V2t)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V5k)))))) \wedge (\forall V6a \in \\ & \quad ty\_2Erealax\_2Ereal. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\ & \quad V6a) V2t)) \Rightarrow (p (ap c\_2Ereal\_topology\_2Ecompact (ap (c\_2Epred\_set\_2EGSPEC \\ & \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V7x \in ty\_2Erealax\_2Ereal. \\ & \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V7x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\ & \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V7x) V1s)) (ap (ap ( \\ & \quad c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) (ap V0f V7x)) V6a))))))))))))) \end{aligned}$$