

thm\_2Ereal\_\_topology\_2EQUOTIENT\_\_MAP\_\_FROM\_\_SUBSET  
(TM-  
RKPHakXzzq2f9z4vsLT7byDPfAmX1NJCM)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

**Definition 8** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}})$$
 (3)

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal$$
 (4)

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal\ ty\_2Erealx\_2Ereal)})$$
 (5)

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal$$
 (6)

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal})$$
 (7)

**Definition 11** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p\ (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 12** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal$ .(ap  $(c\_2Emin\_2E40\ (ap\ p\ x))$  to  $(\lambda x.x \in A \wedge p\ x)$ ).

Let  $c\_2Erealx\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})$$
 (8)

**Definition 13** We define  $c\_2Erealx\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal$ . $\lambda V1T2 \in ty\_2Erealx\_2Ereal$ .(ap  $(c\_2Erealx\_2Etreallt\ (ap\ p\ x))$  to  $(\lambda x.x \in A \wedge p\ x)$ ).

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega$$
 (9)

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum$$
 (10)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega})$$
 (11)

**Definition 14** We define  $c\_2Enum\_2E0$  to be (ap  $c\_2Enum\_2EABS\_num$   $c\_2Enum\_2EZERO\_REP$ ).

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})$$
 (12)

**Definition 15** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 16** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Ebool\_2E\_2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (13)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

**Definition 17** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (15)$$

**Definition 18** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c$

**Definition 19** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 20** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V1s \in$

**Definition 21** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ ($

**Definition 22** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg (p\ V0t)))))) \quad (19) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow \\ & ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). \\ & (\forall V2x \in A\_27c. ((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27c\ A\_27b\ A\_27a) \\ & V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). ((ap\ (ap \\ & (c\_2Epred\_set\_2EIMAGE\ A\_27a\ A\_27a)\ (\lambda V1x \in A\_27a.V1x))\ V0s) = \\ & V0s)) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty\_2Erealx\_2Ereal}). (p\ (ap\ (ap\ c\_2Ereal\_topology\_2Econtinuous\_on \\ & (\lambda V1x \in ty\_2Erealx\_2Ereal.V1x))\ V0s))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}). \\
& (\forall V3t \in (2^{ty\_2Erealax\_2Ereal}).(\forall V4u \in (2^{ty\_2Erealax\_2Ereal}). \\
& (((p (ap (ap (c\_2Ereal\_topology\_2Econtinuous\_on V0f) V2s)) \wedge \\
& ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap ( \\
& ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V0f) V2s)) V3t)) \wedge ((p (ap (ap (c\_2Ereal\_topology\_2Econtinuous\_on \\
& V1g) V3t)) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V1g) V3t)) V4u)) \wedge (\forall V5v \in (2^{ty\_2Erealax\_2Ereal}).((p (ap \\
& (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V5v) V4u)) \Rightarrow \\
& ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) V2s)) (ap (c\_2Epred\_set\_2EGSPEC \\
& ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V6x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V6x) V2s)) (ap (ap ( \\
& c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap (ap (ap (c\_2Ecombin\_2Eo \\
& ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& V1g) V0f) V6x)) V5v)))))) \Leftrightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in \\
& ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) V4u)) \\
& V5v)))))) \Rightarrow (\forall V7v \in (2^{ty\_2Erealax\_2Ereal}).((p (ap ( \\
& ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V7v) V4u)) \Rightarrow ( \\
& (p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap ( \\
& ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Eeuclidean) \\
& V3t)) (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& (\lambda V8x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& 2) V8x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& V8x) V3t)) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V1g V8x)) \\
& V7v)))))) \Leftrightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean) V4u)) V7v)))))))))
\end{aligned} \tag{28}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& (\forall V3u \in (2^{ty\_2Erealax\_2Ereal}).(((p (ap (ap (c\_2Ereal\_topology\_2Econtinuous\_on \\
& \quad V0f) V2t)) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V0f) V2t)) V3u) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& \quad V1s) V2t)) \wedge ((ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) V0f) V1s) = V3u) \wedge (\forall V4v \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V4v) \\
& \quad V3u)) \Rightarrow ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V1s)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V5x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V5x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V5x)) V4v)))))) \Leftrightarrow ( \\
& \quad p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& \quad (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) \\
& \quad V3u)) V4v)))))) \Rightarrow (\forall V6v \in (2^{ty\_2Erealax\_2Ereal}).(( \\
& \quad p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V6v) V3u)) \Rightarrow \\
& \quad ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V2t)) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V7x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V7x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V7x) V2t)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V7x)) V6v)))))) \Leftrightarrow ( \\
& \quad p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& \quad (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) \\
& \quad V3u)) V6v))))))
\end{aligned}$$