

thm_2Ereal__topology_2EQUOTIENT__MAP__OPEN__MAP__EQ (TMHGL3hR1bs6Dmoqn6Y8zp5dFLnNXGcdBk2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (4)$$

Definition 7 We define c_Ebool_EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_Ereal_topology_EDist : \iota$ be given. Assume the following.

$$c_Ereal_topology_EDist \in (ty_Erealax_Ereal^{(ty_Epair_Eprod\ ty_Erealax_Ereal\ ty_Erealax_Ereal)}) \quad (5)$$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (6)$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal}) \quad (7)$$

Definition 8 We define c_Emin_E40 to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p\ (ap\ P\ x)) \mathbf{then} (the\ (\lambda x. x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal. (ap\ (c_Emin_E40\ (ty_Erealax_Ereal\ V0a)))$

Let $c_Erealax_Etrealm_lt : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_lt \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal)}) \quad (8)$$

Definition 10 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal. \lambda V1T2 \in ty_Erealax_Ereal. (c_Erealax_Etrealm_lt\ V0T1\ V1T2)$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \quad (9)$$

Let $ty_Eenum_Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_Eenum_Eenum \quad (10)$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_Eenum_Eenum^{\omega}) \quad (11)$$

Definition 11 We define c_Eenum_E0 to be $(ap\ c_Eenum_EABS_num\ c_Eenum_EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (12)$$

Definition 12 We define c_Ebool_E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_E40\ (ty_Erealax_Ereal\ V0P))))$

Definition 13 We define $c_Ereal_topology_EOpen$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}). (ap\ (c_Ebool_E3F\ (ty_Erealax_Ereal\ V0s)))$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{2^A-27a}})) \quad (14)$$

Definition 14 We define $c_2Ereal_topology_2Euclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax$

Definition 15 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in$

Definition 16 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ ($

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^{A-27a})})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (15)$$

Definition 17 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c$

Definition 18 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 19 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology\ A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ A_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V2t)\ V1s)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (42)$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap \\
& \quad (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal \\
& \quad V0f) V1s)) V2t)) \wedge (\forall V3u \in (2^{ty_2Erealax_2Ereal}).((p (ap \\
& \quad (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V3u) V2t)) \Rightarrow \\
& \quad ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& \quad (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V4x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V4x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) V3u)))))) \Leftrightarrow (\\
& \quad p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap \\
& \quad (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) \\
& \quad V2t)) V3u)))))) \Rightarrow ((\forall V5k \in (2^{ty_2Erealax_2Ereal}).((p (\\
& \quad ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap (\\
& \quad c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) \\
& \quad V1s)) V5k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V2t)) (ap (ap (c_2Epred_set_2EIMAGE \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V5k)))))) \Leftrightarrow (\forall V6k \in \\
& \quad (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V1s)) \\
& \quad V6k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V7x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V7x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V7x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V7x)) (ap (ap (c_2Epred_set_2EIMAGE \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V6k)))))))))
\end{aligned}$$