

thm_2Ereal__topology_2EQUOTIENT__MAP__RESTRICT (TMKE12xeuosbbnXQBXWHsjva69kSWz3VYEm)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p \text{ (ap } P \ x)) \text{ then (the } (\lambda x.x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.\text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $\text{ty_2Epair_2Eprod} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \quad (1)$$

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(\text{ap } V0P \text{ (ap } (\text{c_2Emin_2E_40 } A_{27a} \ P))$

Let $\text{ty_2Etopology_2Etopology} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Etopology_2Etopology } A0) \quad (2)$$

Let $\text{ty_2Erealax_2Ereal} : \iota$ be given. Assume the following.

$$\text{nonempty ty_2Erealax_2Ereal} \quad (3)$$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o } (p \Rightarrow p \ Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}} \ P))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.(\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}} \ P))$

Let $\text{c_2Epair_2EABS_prod} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.\text{nonempty } A_{27a} \Rightarrow \forall A_{27b}.\text{nonempty } A_{27b} \Rightarrow \text{c_2Epair_2EABS_prod } A_{27a} \ A_{27b} \in ((\text{ty_2Epair_2Eprod } A_{27a} \ A_{27b}))^{((2^{A_{27b}})^{A_{27a}})} \quad (4)$$

Definition 8 We define $c_Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (7)$$

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E_40\ (ty$

Let $c_2Erealax_2Ereal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \quad (8)$$

Definition 10 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (11)$$

Definition 12 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 13 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax})$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{A_27a})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (13)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2^{A_27b})}) \quad (14)$$

Definition 14 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2EBIGUNION) P)$

Definition 15 We define $c_2Etopology_2Etopospace$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopospace)$

Definition 16 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E7E))$

Definition 18 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EDIFF) s)$

Definition 19 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2ESUBSET) s)$

Definition 20 We define $c_2Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopospace)$

Definition 21 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E21\ 2) s)$

Let $c_2Etopology_2Etopospace : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopospace\ A_27a \in ((ty_2Etopology_2Etopospace\ A_27a)^{(2^{(2^{A_27a})})}) \quad (15)$$

Definition 22 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopospace\ ty_2Erealax))$

Definition 23 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EINTER) s)$

Definition 24 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopospace)$

Definition 25 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b}).(ap (c_2Epred_set_2EIMAGE) f)$

Definition 26 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21\ 2) (\lambda V2t \in 2.V2t))))$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V1x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ &(p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ &(((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ &(((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge ((p \ V0t) \vee \\ &(p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ &True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ &(p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} &((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ &((\neg False) \Leftrightarrow True))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\begin{aligned} &\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ &A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ &(p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ &p \ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (36)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))) \quad (38)$$

Assume the following.

$$(\forall V0r \in 2. (\forall V1p \in 2. (\forall V2q \in 2. (((p\ V1p) \wedge (p\ V2q)) \Rightarrow (p\ V0r)) \Leftrightarrow ((p\ V2q) \Rightarrow ((p\ V1p) \Rightarrow (p\ V0r)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (43)$$

Assume the following.

$$(\forall V0P \in 2. (\forall V1Q \in 2. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \Rightarrow ((p\ V0P) \Rightarrow (p\ V1Q)))))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ & A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((p\ (\\ & ap\ (ap\ (c_2Etopology_2Eopen_in\ A_27a)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\ & A_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a) \\ & V2t)\ V1s)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ & A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((p\ (\\ & ap\ (ap\ (c_2Etopology_2Eclosed_in\ A_27a)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\ & A_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a) \\ & V2t)\ V1s)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V2u)) \\
& \quad V0s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad V0s) V1t)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad V1t) V2u)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad \quad c_2Ereal_topology_2Euclidean) V1t)) V0s))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Etopology_2Eclosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V2u)) \\
& \quad V0s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad V0s) V1t)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad V1t) V2u)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad \quad c_2Ereal_topology_2Euclidean) V1t)) V0s))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V1t)) \\
& \quad V0s)) \wedge (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad \quad c_2Ereal_topology_2Euclidean) V2u)) V1t)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V2u)) \\
& \quad \quad V0s))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Etopology_2Eclosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V1t)) \\
& \quad V0s)) \wedge (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V2u)) V1t)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V2u)) \\
& \quad \quad \quad V0s))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f) V1s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s)) V2t)) \wedge (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V2t)) V3u)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V1s)) \\
& \quad (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad (\lambda V4x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad 2) V4x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad \quad V4x) V1s)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) \\
& \quad \quad \quad V3u))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f) V1s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s)) V2t)) \wedge (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V2t)) V3u)))) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V1s)) \\
& \quad (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad (\lambda V4x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad 2) V4x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V4x) V1s)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) \\
& \quad V3u))))))))))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& \quad (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap \\
& \quad (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s)) V2t)) \wedge (\forall V3u \in (2^{ty_2Erealax_2Ereal}).((p (ap \\
& \quad (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V3u) V2t)) \Rightarrow \\
& \quad ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& \quad (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V4x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V4x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) V3u)))))) \Leftrightarrow (\\
& \quad p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap \\
& \quad (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) \\
& \quad V2t)) V3u)))))) \Rightarrow (p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f) V1s)))))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap (\\
& \quad ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s)) V2t)) \Rightarrow ((\forall V3u \in (2^{ty_2Erealax_2Ereal}).((p (\\
& \quad ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V3u) V2t)) \Rightarrow \\
& \quad ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap \\
& \quad (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V4x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V4x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) V3u)))))) \Leftrightarrow (\\
& \quad p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap \\
& \quad (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) \\
& \quad V2t)) V3u)))) \Leftrightarrow (\forall V5u \in (2^{ty_2Erealax_2Ereal}).((p (ap \\
& \quad (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V5u) V2t)) \Rightarrow \\
& \quad ((p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) (\\
& \quad ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V6x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V6x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V6x)) V5u)))))) \Leftrightarrow (\\
& \quad p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) (ap \\
& \quad (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V2t)) V5u))))))))) \tag{54}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{57}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{58}
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p \ V0A)) \Rightarrow False) \Rightarrow ((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (59)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(\\ &p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\ &((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\ &(\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\ &((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\ &\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\ &(p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \end{aligned} \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \quad (69)$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& \quad (\forall V3c \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Epred_set_2ESUBSET \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal) V0f) V1s)) V2t)) \wedge ((\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V4u) \\
& \quad V2t)) \Rightarrow ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V1s)) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V5x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V5x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V5x)) V4u)))))) \Leftrightarrow (\\
& \quad p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) (ap (ap \\
& \quad (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) \\
& \quad V2t)) V4u)))))) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V2t)) V3c)) \vee (p (ap (ap (c_2Etopology_2Eclosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V2t)) \\
& \quad V3c)))))) \Rightarrow (\forall V6u \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Epred_set_2ESUBSET \\
& \quad ty_2Erealax_2Ereal) V6u) V3c)) \Rightarrow ((p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) (ap (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal ty_2Erealax_2Ereal) (\lambda V7x \in ty_2Erealax_2Ereal. \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal 2) V7x) (ap (ap c_2Ebool_2E_2F_5C \\
& \quad (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V7x) V1s)) (ap (ap (\\
& \quad c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V7x)) V3c)))))) (ap \\
& \quad (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad (\lambda V8x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad 2) V8x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V8x) (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad (\lambda V9x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad 2) V9x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V9x) V1s)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V9x)) \\
& \quad V3c)))))) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f \\
& \quad V8x)) V6u)))))) \Leftrightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Euclidean) V3c)) V6u)))))))))
\end{aligned}$$