thm_2Ereal__topology_2EREAL__HAUSDIST__LE (TMM-bgBD5GgHygCTECHZDKGh6bGSKj6rhvug)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if } (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then } (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{2}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b}$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{4}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 6 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{7}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{9}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{10}$$

Definition 7 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{11}$$

Definition 8 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 12 *We define* c_2Ereal_2Esup *to be* $\lambda V0P \in (2^{ty\_2Erealax\_2Ereal}).(ap$ (c_2Emin_2E_40 ty_2Ereal

Definition 13 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 16 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 17 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 18 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{12}$$

Definition 19 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{13}$$

Definition 20 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Let $c\_2Ereal\_\_topology\_2Esetdist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Esetdist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal} \tag{14}$$

Definition 21 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_$

Definition 22 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Ereal\_\_topology\_2Ehausdist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2Ehausdist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Er} \tag{15}$$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t)))))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{22}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\wedge(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\wedge((p\ V0A)\vee(p\ V2C))))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B)\wedge(p\ V2C))\vee(p\ V0A))\Leftrightarrow(((p\ V1B)\vee(p\ V0A))\wedge((p\ V2C)\vee(p\ V0A))))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\Rightarrow(p\ V1B))\Leftrightarrow((\neg(p\ V0A))\vee(p\ V1B))))) \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{33}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v)\Rightarrow(p\ (ap\ V0f\ V2x))))\Leftrightarrow(p\ (ap\ V0f\ V1v))))) \tag{36}$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1b \in ty\_2Erealax\_2Ereal.(((\neg(V0s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))\wedge(\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V2x)\ V0s))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V2x)\ V1b)))))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Ereal\_2Esup}\ V0s))\ V1b))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) \Leftrightarrow ((V0s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) \wedge (V1t = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27a}).((\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V1s)\ V2t))) \Rightarrow (p\ (ap\ V0P\ V3x)))) \Leftrightarrow ((\forall V4x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V4x)\ V1s)) \Rightarrow (p\ (ap\ V0P\ V4x)))) \wedge (\forall V5x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V5x)\ V2t)) \Rightarrow (p\ (ap\ V0P\ V5x))))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow (\forall V0Q \in (2^{A\_27b}).((\forall V1P \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}).((\forall V3z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V3z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ A\_27a)\ (\lambda V4x \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x))))))\Rightarrow (p\ (ap\ V0Q\ V3z))))\Leftrightarrow (\forall V5x \in A\_27a.((p\ (ap\ V1P\ V5x))\Rightarrow (p\ (ap\ V0Q\ (ap\ V2f\ V5x))))))))\land ((\forall V6P \in ((2^{A\_27d})^{A\_27c}).(\forall V7f \in ((A\_27b^{A\_27d})^{A\_27c}).((\forall V8z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V8z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27c\ A\_27d))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27c\ A\_27d\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V9x \in A\_27c.(\lambda V10y \in A\_27d.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))))) \Rightarrow (p\ (ap\ V0Q\ V8z))))\Leftrightarrow (\forall V11x \in A\_27c.(\forall V12y \in A\_27d.((p\ (ap\ (ap\ V6P\ V11x)\ V12y))\Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y))))))))) \land (\forall V13P \in (((2^{A\_27g})^{A\_27f})^{A\_27e}).(\forall V14f \in (((A\_27b^{A\_27g})^{A\_27f})^{A\_27e}). ((\forall V15z \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V15z)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27e\ (ty\_2Epair\_2Eprod\ A\_27f\ A\_27g)\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V16w \in A\_27e.(ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27f\ A\_27g\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V17x \in A\_27f.(\lambda V18y \in A\_27g.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x)\ V18y)))))))))) \Rightarrow (p\ (ap\ V0Q\ V15z))))\Leftrightarrow (\forall V19w \in A\_27e.(\forall V20x \in A\_27f.(\forall V21y \in A\_27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y))\Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))))) \tag{44}$$

Assume the following.

$$
\begin{gathered}
(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
((ap\ c\_2Ereal\_\_topology\_2Ehausdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (\\
2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t)) = (\\
ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C} \\
(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{ty\_2Erealax\_2Ereal})) \\
(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC \\
ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V2x \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ (ap\ c\_2Ereal\_\_topology\_2Esetdist \\
(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})) \\
(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V2x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY} \\
ty\_2Erealax\_2Ereal)))\ V1t)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal) \\
V2x)\ V0s)))))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal)\ (\lambda V3y \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
ty\_2Erealax\_2Ereal\ 2)\ (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap \\
(ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})) \\
(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V3y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY} \\
ty\_2Erealax\_2Ereal)))\ V0s)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal) \\
V3y)\ V1t))))))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))) \\
(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ty\_2Erealax\_2Ereal)\ (\lambda V4b \in ty\_2Erealax\_2Ereal. \\
(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ ty\_2Erealax\_2Ereal)\ (\lambda V5d \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal) \\
V5d)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Erealax\_2Ereal)\ (ap\ (\\
c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) \\
(\lambda V6x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal \\
2)\ (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
(2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT} \\
ty\_2Erealax\_2Ereal)\ V6x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))) \\
V1t)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V6x)\ V0s))))) \\
(ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) \\
(\lambda V7y \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal \\
2)\ (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
(2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT} \\
ty\_2Erealax\_2Ereal)\ V7y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))) \\
V0s)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V7y)\ V1t))))))) \\
(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V5d)\ V4b))))))))\ (ap\ \mathsf{c\_2Ereal\_2Esup} \\
(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC \\
ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ (\lambda V8x \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ 2)\ (ap\ c\_2Ereal\_\_topology\_2Esetdist \\
(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})) \\
(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V8x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY} \\
ty\_2Erealax\_2Ereal)))\ V1t)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal) \\
V8x)\ V0s)))))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal \\
ty\_2Erealax\_2Ereal)\ (\lambda V9y \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C} \\
ty\_2Erealax\_2Ereal\ 2)\ (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap \\
(ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})) \\
(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V9y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY} \\
ty\_2Erealax\_2Ereal)))\ V0s)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal) \\
V9y)\ V1t)))))))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))))
\end{gathered}
\tag{45}
$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{52}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{53}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{57}$$

Theorem 1

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). (\forall V2b \in ty\_2Erealax\_2Ereal.(((\neg(V0s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))) \wedge ((\neg(V1t = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal))) \wedge ((\forall V3x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V3x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))\ V1t)))\ V2b)))) \wedge (\forall V4y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Erealax\_2Ereal)\ V4y)\ V1t)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_\_topology\_2Esetdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Erealax\_2Ereal)\ V4y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Erealax\_2Ereal)))\ V0s)))\ V2b))))))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_\_topology\_2Ehausdist\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t)))\ V2b))))))$$