

thm_2Ereal__topology_2EREAL__LE__HAUSDIST
(TMdAG-
BcRG8CQQWGJCR9Pmpo1TDW7zRR1ek)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$.
Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \tag{5}$$

Definition 6 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 10 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21) 2) (\lambda V2t2 \in 2.V2t2))))$

Definition 13 We define c_2Ereal_2Emax to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (6)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (7)$$

Definition 14 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 17 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Emin_2E_40 ty_2Ereal$

Definition 18 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap\ V1f\ V0x)))$

Definition 19 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_21\ 2)\ V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (12)$$

Definition 21 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2E_21\ 2)\ (c_2Ebool_2E_21\ 2)\ V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \end{aligned} \quad (13)$$

Definition 22 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A-27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (c_2Ebool_2E_21\ 2)\ V0x\ V1s))$

Let $c_2Ereal_topology_2Esetdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Esetdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))})^{(2^{ty_2Erealax_2Ereal})}) \quad (14)$$

Definition 23 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (c_2Ebool_2E_21\ 2)\ V0s\ V1t))$

Let $c_2Ereal_topology_2Ehausdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Ehausdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))})^{(2^{ty_2Erealax_2Ereal})}) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ A_27a. (p\ V0t) \Leftrightarrow (p\ V1x)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))) \quad (24)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (V0x = V0x)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2)))) \quad (29)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in (2^{A-27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A-27a}).((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$(\forall V0y \in 2. (\forall V1x \in 2. (((p\ V0y) \Rightarrow (p\ V1x)) \Leftrightarrow ((\neg(p\ V1x)) \Rightarrow (\neg(p\ V0y)))))) \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in \\
& A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\
& ap\ V0f\ V1v))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1a \in ty_2Erealax_2Ereal. \\
& (\forall V2b \in ty_2Erealax_2Ereal. (\forall V3y \in ty_2Erealax_2Ereal. \\
& (((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V3y)\ V0s)) \wedge ((\\
& p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1a)\ V3y)) \wedge (\forall V4x \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V4x)\ V0s)) \Rightarrow (p\ (\\
& ap\ (ap\ c_2Ereal_2Ereal_lte\ V4x)\ V2b)))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& V1a)\ (ap\ c_2Ereal_2Esup\ V0s)))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& A_27a\ A_27b)\ V0f)) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\neg (p (ap (ap (c_2Ebool_2EIN A_{.27a}) V0x) (c_2Epred_set_2EEMPTY A_{.27a})))))) \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (\forall V1t \in \\ (2^{A_{.27a}}). (\forall V2x \in A_{.27a}. ((p (ap (ap (c_2Ebool_2EIN A_{.27a}) \\ V2x) (ap (ap (c_2Epred_set_2EUNION A_{.27a}) V0s) V1t))) \Leftrightarrow ((p (ap \\ (ap (c_2Ebool_2EIN A_{.27a}) V2x) V0s)) \vee (p (ap (ap (c_2Ebool_2EIN \\ A_{.27a}) V2x) V1t)))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (\forall V1t \in \\ (2^{A_{.27a}}). (((ap (ap (c_2Epred_set_2EUNION A_{.27a}) V0s) V1t) = \\ (c_2Epred_set_2EEMPTY A_{.27a})) \Leftrightarrow ((V0s = (c_2Epred_set_2EEMPTY \\ A_{.27a})) \wedge (V1t = (c_2Epred_set_2EEMPTY A_{.27a})))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}). (\forall V1s \in \\ (2^{A_{.27a}}). (\forall V2t \in (2^{A_{.27a}}). ((\forall V3x \in A_{.27a}. ((p (\\ ap (ap (c_2Ebool_2EIN A_{.27a}) V3x) (ap (ap (c_2Epred_set_2EUNION \\ A_{.27a}) V1s) V2t))) \Rightarrow (p (ap V0P V3x)))) \Leftrightarrow ((\forall V4x \in A_{.27a}. ((p \\ (ap (ap (c_2Ebool_2EIN A_{.27a}) V4x) V1s)) \Rightarrow (p (ap V0P V4x)))) \wedge (\forall V5x \in \\ A_{.27a}. ((p (ap (ap (c_2Ebool_2EIN A_{.27a}) V5x) V2t)) \Rightarrow (p (ap V0P V5x)))))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} (\forall V0z \in \text{ty_2Erealax_2Ereal}. (\forall V1x \in \text{ty_2Erealax_2Ereal}. \\ (\forall V2y \in \text{ty_2Erealax_2Ereal}. ((p (ap (ap (c_2Ereal_2Ereal_lte \\ V0z) (ap (ap (c_2Ereal_2Ereal_max V1x) V2y))) \Leftrightarrow ((p (ap (ap (c_2Ereal_2Ereal_lte \\ V0z) V1x)) \vee (p (ap (ap (c_2Ereal_2Ereal_lte V0z) V2y)))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\
& A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow \forall A_27g.nonempty\ A_27g \Rightarrow \\
& (\forall V0Q \in (2^{A_27b}). (\forall V1P \in (2^{A_27a}). (\forall V2f \in \\
& (A_27b^{A_27a}). (\forall V3z \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27b)\ V3z)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27b\ A_27a)\ (\lambda V4x \in \\
& A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\
& (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A_27a. ((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\
& (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A_27d})^{A_27c}). (\forall V7f \in \\
& ((A_27b^{A_27d})^{A_27c}). (\forall V8z \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& A_27b)\ V8z)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27b\ (ty_2Epair_2Eprod \\
& A_27c\ A_27d))\ (ap\ (c_2Epair_2EUNCURRY\ A_27c\ A_27d\ (ty_2Epair_2Eprod \\
& A_27b\ 2))\ (\lambda V9x \in A_27c. (\lambda V10y \in A_27d. (ap\ (ap\ (c_2Epair_2E_2C \\
& A_27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\
& (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A_27c. (\forall V12y \in A_27d. ((p \\
& (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A_27g})^{A_27f})^{A_27e}). (\forall V14f \in (((A_27b^{A_27g})^{A_27f})^{A_27e}). \\
& (\forall V15z \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V15z)\ (\\
& ap\ (c_2Epred_set_2EGSPEC\ A_27b\ (ty_2Epair_2Eprod\ A_27e\ (ty_2Epair_2Eprod \\
& A_27f\ A_27g)))\ (ap\ (c_2Epair_2EUNCURRY\ A_27e\ (ty_2Epair_2Eprod \\
& A_27f\ A_27g)\ (ty_2Epair_2Eprod\ A_27b\ 2))\ (\lambda V16w \in A_27e. (ap \\
& (c_2Epair_2EUNCURRY\ A_27f\ A_27g\ (ty_2Epair_2Eprod\ A_27b\ 2)) \\
& (\lambda V17x \in A_27f. (\lambda V18y \in A_27g. (ap\ (ap\ (c_2Epair_2E_2C\ A_27b \\
& 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x) \\
& V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A_27e. (\forall V20x \in \\
& A_27f. (\forall V21y \in A_27g. ((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\
& (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap\ c_2Ereal_topology_2Ehausdist\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& \quad 2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ V0s)\ V1t))) = (\\
& \quad ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Erealax_2Ereal)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad \quad (ap\ c_2Ebool_2E_7E\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V2x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V2x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V2x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V3y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V3y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V3y)\ V1t))))\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad (ap\ (c_2Ebool_2E_3F\ ty_2Erealax_2Ereal)\ (\lambda V4b \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (c_2Ebool_2E_21\ ty_2Erealax_2Ereal)\ (\lambda V5d \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad \quad V5d)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (\\
& \quad \quad \quad c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad \quad (\lambda V6x \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad \quad \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V6x)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad \quad \quad V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V6x)\ V0s)))) \\
& \quad \quad (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad \quad (\lambda V7y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad \quad \quad 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad \quad \quad ty_2Erealax_2Ereal)\ V7y)\ (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \\
& \quad \quad \quad V0s))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V7y)\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V7y)\ V1t))))\ (ap\ (ap\ (c_2Ereal_2Ereal_lte\ V5d)\ V4b))))\ (ap\ c_2Ereal_2Esup \\
& (ap\ (ap\ (c_2Epred_set_2EUNION\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V8x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist \\
& \quad \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V8x)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V1t))))\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V8x)\ V0s))))\ (ap\ (c_2Epred_set_2EGSPEC\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ (\lambda V9y \in ty_2Erealax_2Ereal.(ap\ (ap\ (c_2Epair_2E_2C \\
& \quad ty_2Erealax_2Ereal\ 2)\ (ap\ c_2Ereal_topology_2Esetdist\ (ap \\
& \quad \quad (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal})) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Erealax_2Ereal)\ V9y)\ (c_2Epred_set_2EEMPTY \\
& \quad ty_2Erealax_2Ereal))))\ V0s))))\ (\emptyset\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal \\
& \quad V9y)\ V1t))))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \\
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (63)$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2a \in ty_2Erealax_2Ereal. (\forall V3b \in ty_2Erealax_2Ereal. \\
& (\forall V4c \in ty_2Erealax_2Ereal. (\forall V5z \in ty_2Erealax_2Ereal. \\
& (((\neg(V0s = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \wedge ((\\
& \neg(V1t = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \wedge ((\forall V6x \in \\
& ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V6x) V0s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V6x) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V1t))) V3b)))) \wedge ((\forall V7y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V7y) V1t)) \Rightarrow (p (\\
& ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V7y) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V0s))) V4c)))) \wedge (((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V5z) V0s)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V2a) (ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) (ap (ap (c_2Epred_set_2EINSERT \\
& ty_2Erealax_2Ereal) V5z) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))) \\
& V1t)))))) \vee ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5z) \\
& V1t)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V2a) (ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V5z) (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal))) V0s))))))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V2a) (ap c_2Ereal_topology_2Ehausdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V0s) V1t)))))))))
\end{aligned}$$