

# thm\_2Ereal\_topology\_2EREAL\_LE\_SETDIST (TMJoemP4Lch5LmDVD8axk2qundH3Hch7vFP)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p (ap P x))$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty\_2Emin\_2E\_40$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \tag{5}$$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$

**Definition 11** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 12** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)}) \quad (6)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (7)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (8)$$

**Definition 13** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})^{A\_27b})$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \end{aligned} \quad (9)$$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t)))$

**Definition 15** We define  $c\_2Eiterate\_2Einf$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Emin\_2E\_40 ty\_2Erealax\_2Ereal))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (10)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (11)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (12)$$

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 17** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (14)$$

**Definition 19** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2$

Let  $c\_2Ereal\_topology\_2Esetdist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2Esetdist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})})} \quad (15)$$

**Definition 20** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \wedge \\ ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A.27a) \ c\_2Ebool\_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A.27a) \ c\_2Ebool\_2EF) \\
& V0t1) \ V1t2) = V1t2)))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in \\
& A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in \\
& A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q))) \Leftrightarrow (\exists V3x \in \\ A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p \ (ap \ V0P \ V3x))) \wedge (p \ V1Q)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A-27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ (p \ V0A)))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\ p \ V0A)) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ 2.(((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1a \in \\ A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ ( \\ ap \ V0P \ V1a)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ \forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0s \in (2^{ty.2Erealax.2Ereal}).(((\neg(V0s = (c.2Epred\_set.2EEMPTY \\ ty.2Erealax.2Ereal)))) \wedge (\exists V1b \in ty.2Erealax.2Ereal.(\forall V2x \in \\ ty.2Erealax.2Ereal.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealax.2Ereal \\ V2x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ V1b)\ V2x)))))) \Rightarrow ((\forall V3x \in \\ ty.2Erealax.2Ereal.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealax.2Ereal \\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ (ap\ c.2Eiterate.2Einf \\ V0s)\ V3x)))))) \wedge (\forall V4b \in ty.2Erealax.2Ereal.((\forall V5x \in \\ ty.2Erealax.2Ereal.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealax.2Ereal \\ V5x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal\_lte\ V4b)\ V5x)))))) \Rightarrow (p\ (ap \\ (ap\ c.2Ereal.2Ereal\_lte\ V4b)\ (ap\ c.2Eiterate.2Einf\ V0s)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\ A.27b.(((ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y) = (ap\ (ap \\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}).(\forall V1x \in \\ A.27a.(\forall V2y \in A.27b.((ap\ (ap\ (c.2Epair.2EUNCURRY\ A.27a \\ A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V1x)\ V2y)) = \\ (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in (2^{(ty.2Epair.2Eprod\ A.27a\ A.27b)}).((\exists V1p \in \\ (ty.2Epair.2Eprod\ A.27a\ A.27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p\_1 \in \\ A.27a.(\exists V3p\_2 \in A.27b.(p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair.2E.2C \\ A.27a\ A.27b)\ V2p\_1)\ V3p\_2)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in \\ & A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap \\ & (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty \\ & A\_27e \Rightarrow \forall A\_27f.nonempty\ A\_27f \Rightarrow \forall A\_27g.nonempty\ A\_27g \Rightarrow \\ & \quad (\forall V0Q \in (2^{A\_27b}).(\forall V1P \in (2^{A\_27a}).(\forall V2f \in \\ & \quad (A\_27b^{A\_27a}).(\forall V3z \in A\_27b.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27b)\ V3z)\ (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27b\ A\_27a)\ (\lambda V4x \in \\ & A\_27a.(ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\ & \quad (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A\_27a.((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\ & \quad (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A\_27d})^{A\_27c}).(\forall V7f \in \\ & \quad ((A\_27b^{A\_27d})^{A\_27c}).(\forall V8z \in A\_27b.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27b)\ V8z)\ (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod \\ & \quad A\_27c\ A\_27d))\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27c\ A\_27d\ (ty\_2Epair\_2Eprod \\ & \quad A\_27b\ 2))\ (\lambda V9x \in A\_27c.(\lambda V10y \in A\_27d.(ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\ & \quad (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A\_27c.(\forall V12y \in A\_27d.((p \\ & (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\ & \quad (\forall V13P \in (((2^{A\_27g})^{A\_27f})^{A\_27e}).(\forall V14f \in (((A\_27b^{A\_27g})^{A\_27f})^{A\_27e}). \\ & \quad ((\forall V15z \in A\_27b.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V15z)\ ( \\ & ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27b\ (ty\_2Epair\_2Eprod\ A\_27e\ (ty\_2Epair\_2Eprod \\ & \quad A\_27f\ A\_27g)))\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27e\ (ty\_2Epair\_2Eprod \\ & \quad A\_27f\ A\_27g)\ (ty\_2Epair\_2Eprod\ A\_27b\ 2))\ (\lambda V16w \in A\_27e.(ap \\ & \quad (c\_2Epair\_2EUNCURRY\ A\_27f\ A\_27g\ (ty\_2Epair\_2Eprod\ A\_27b\ 2)) \\ & \quad (\lambda V17x \in A\_27f.(\lambda V18y \in A\_27g.(ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b \\ & \quad 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x) \\ & \quad V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A\_27e.(\forall V20x \in \\ & A\_27f.(\forall V21y \in A\_27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\ & \quad (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y))))))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (p (ap (ap (c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C \\
& ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0x) V1y))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((ap c\_2Ereal\_topology\_2Esetdist (ap (ap (c\_2Epair\_2E\_2C (2^{ty\_2Erealax\_2Ereal}) \\
& (2^{ty\_2Erealax\_2Ereal})) V0s) V1t)) = (ap (ap (ap (c\_2Ebool\_2ECOND \\
& ty\_2Erealax\_2Ereal) (ap (ap c\_2Ebool\_2E\_5C\_2F (ap (ap (c\_2Emin\_2E\_3D \\
& (2^{ty\_2Erealax\_2Ereal})) V0s) (c\_2Epred\_set\_2EEMPTY ty\_2Erealax\_2Ereal))) \\
& (ap (ap (c\_2Emin\_2E\_3D (2^{ty\_2Erealax\_2Ereal})) V1t) (c\_2Epred\_set\_2EEMPTY \\
& ty\_2Erealax\_2Ereal)))))) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& (ap c\_2Eiterate\_2Einf (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal \\
& (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)) \\
& (ap (c\_2Epair\_2EUNCURRY ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal \\
& (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal 2)) (\lambda V2x \in ty\_2Erealax\_2Ereal. \\
& (\lambda V3y \in ty\_2Erealax\_2Ereal. (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& 2) (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V2x) V3y))) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap \\
& (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V2x) V0s)) (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V3y) V1t))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg( \\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{62}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& (\forall V2d \in ty\_2Erealax\_2Ereal. (((\neg(V0s = (c\_2Epred\_set\_2EEMPTY \\
& ty\_2Erealax\_2Ereal))) \wedge ((\neg(V1t = (c\_2Epred\_set\_2EEMPTY \ ty\_2Erealax\_2Ereal)))) \wedge \\
& (\forall V3x \in ty\_2Erealax\_2Ereal. (\forall V4y \in ty\_2Erealax\_2Ereal. \\
& (((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ ty\_2Erealax\_2Ereal) \ V3x) \ V0s)) \wedge (p \\
& (ap \ (ap \ (c\_2Ebool\_2EIN \ ty\_2Erealax\_2Ereal) \ V4y) \ V1t))) \Rightarrow (p \ (ap \\
& (ap \ c\_2Ereal\_2Ereal\_lte \ V2d) \ (ap \ c\_2Ereal\_topology\_2EDist \\
& (ap \ (ap \ (c\_2Epair\_2E\_2C \ ty\_2Erealax\_2Ereal \ ty\_2Erealax\_2Ereal) \\
& V3x) \ V4y)))))) \Rightarrow (p \ (ap \ (ap \ c\_2Ereal\_2Ereal\_lte \ V2d) \ (ap \ c\_2Ereal\_topology\_2Esetdist \\
& (ap \ (ap \ (c\_2Epair\_2E\_2C \ (2^{ty\_2Erealax\_2Ereal}) \ (2^{ty\_2Erealax\_2Ereal}) \\
& V0s) \ V1t))))))
\end{aligned}$$