

thm_2Ereal__topology_2ESESEQUENCE__INFINITE__LEMMA (TMV4ve5BgyuU1bz8cfwz4anQzXAhLqz5XNZ)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (2)$$

Definition 6 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P) V0P))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A.27a}).\lambda V1s \in$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (5)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (6)$$

Definition 12 We define c_2Ebool_2E21 to be $(ap (c_2Ebool_2E21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 13 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Let $c_2Eenum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EREP_num \in (\omega^{ty_2Eenum_2Eenum}) \quad (7)$$

Let $c_2Eenum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Eenum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Let $c_2Eenum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EABS_num \in (ty_2Eenum_2Eenum^{\omega}) \quad (9)$$

Definition 14 We define c_2Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum.(ap\ c_2Eenum_2EABS_num$

Definition 15 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$).

Definition 16 We define c_2Ebool_2E3F to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ V0P (ap (c_2Emin_2E40$

Definition 17 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 18 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21\ 2)) (\lambda V2t \in$

Definition 19 We define $c_2Earithmetic_2E3C_3D$ to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Let $c_2Eenum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Eenum_2EZERO_REP \in \omega \quad (10)$$

Definition 20 We define c_2Eenum_2E0 to be $(ap\ c_2Eenum_2EABS_num\ c_2Eenum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (13)$$

Definition 21 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal_REP_CLASS\ a))$

Let $c_2Erealax_2Etrealm : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Definition 22 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal_lt\ T1\ T2)$

Definition 23 We define $c_2Earithmic_2E3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Earithmic_2E3E\ m\ n)$

Definition 24 We define $c_2Earithmic_2E3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Earithmic_2E3E_3D\ m\ n)$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (15)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net\ A_27a \\ A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (16)$$

Definition 25 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Ereal_topology_2Enet))$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet\ A_27a)}) \quad (17)$$

Definition 26 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 27 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Definition 28 We define $c_2Ereal_topology_2E2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^{A_27a})$

Definition 29 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_2Ereal_2Ereal_lte\ x\ y)$

Definition 30 We define $c_2Eiterate_2Einf$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal_REP_CLASS\ s))$

Definition 31 We define `c_2Epred_set_2EEMPTY` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 32 We define `c_2Epred_set_2EINSERT` to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2E21 (2^{A_27a})))$.

Definition 33 We define `c_2Epred_set_2EFINITE` to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E21 (2^{A_27a})))$.

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V0m))) \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg (p V0t)))) \quad (22)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V1x)))) \quad (23)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p V0t) \Leftrightarrow (p V1x)))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\neg(\forall V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\neg(\exists V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (39)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B)))))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\exists V2y \in A_27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}).(\forall V4x \in A_27a.(p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in ((ty_2Epair_2Eprod A_27a 2)^{A_27b}).(\forall V1v \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1v) (ap (c_2Epred_set_2EGSPEC A_27a A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap (ap (c_2Epair_2E_2C A_27a 2) V1v) c_2Ebool_2ET) = (ap V0f V2x)))))) \quad (52)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((\exists V1x \in A_27a.(p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0s))) \Leftrightarrow (\neg(V0s = (c_2Epred_set_2EEMPTY A_27a)))))) \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0y \in A_27b.(\forall V1s \in (2^{A_27a}).(\forall V2f \in (A_27b^{A_27a}).((p (ap (ap (c_2Ebool_2EIN A_27b) V0y) (ap (ap (c_2Epred_set_2EIMAGE A_27a A_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_27a.((V0y = (ap V2f V3x)) \wedge (p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V1s))))))) \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0s \in (2^{A.27a}).((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a) \\ & \quad V0s)) \Rightarrow (\forall V1f \in (A.27b^{A.27a}).(p\ (ap\ (c.2Epred_set.2EFINITE \\ & \quad A.27b)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27b)\ V1f)\ V0s)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal.(\forall V1y \in ty.2Erealax.2Ereal. \\ & ((\neg(p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ V0x)\ V1y)))) \Leftrightarrow (p\ (ap\ (ap\ c.2Erealax.2Ereal_lt \\ & \quad V1y)\ V0x)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal.(\forall V1y \in ty.2Erealax.2Ereal. \\ & ((\neg(V0x = V1y)) \Rightarrow (p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal.2Ereal_of_num \\ & \quad c.2Enum.2E0))\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C \\ & \quad ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal)\ V0x)\ V1y)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (ty.2Erealax.2Ereal^{ty.2Enum.2Enum}).(\forall V1l \in \\ & \quad ty.2Erealax.2Ereal.((p\ (ap\ (ap\ (ap\ (c.2Ereal_topology.2E.2D.2D.3E \\ & \quad ty.2Enum.2Enum)\ V0s)\ V1l)\ c.2Ereal_topology.2Esequentially)) \Leftrightarrow \\ & \quad (\forall V2e \in ty.2Erealax.2Ereal.((p\ (ap\ (ap\ c.2Erealax.2Ereal_lt \\ & \quad (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))\ V2e)) \Rightarrow (\exists V3N \in \\ & \quad ty.2Enum.2Enum.(\forall V4n \in ty.2Enum.2Enum.((p\ (ap\ (ap\ c.2Earithmetic.2E.3C.3D \\ & \quad V3N)\ V4n)) \Rightarrow (p\ (ap\ (ap\ c.2Erealax.2Ereal_lt\ (ap\ c.2Ereal_topology.2EDist \\ & \quad (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealax.2Ereal\ ty.2Erealax.2Ereal) \\ & \quad (ap\ V0s\ V4n))\ V1l)))\ V2e)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty.2Erealax.2Ereal}).(((p\ (ap\ (c.2Epred_set.2EFINITE \\ & \quad ty.2Erealax.2Ereal)\ V0s)) \wedge (\neg(V0s = (c.2Epred_set.2EEMPTY\ ty.2Erealax.2Ereal)))) \Rightarrow \\ & \quad ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealax.2Ereal)\ (ap\ c.2Eiterate.2Einf \\ & \quad V0s))\ V0s)) \wedge (\forall V1x \in ty.2Erealax.2Ereal.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & \quad ty.2Erealax.2Ereal)\ V1x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ c.2Ereal.2Ereal_lte \\ & \quad (ap\ c.2Eiterate.2Einf\ V0s))\ V1x)))))) \end{aligned} \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (74)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1l \in \\ & ty_2Erealax_2Ereal.(((\forall V2n \in ty_2Enum_2Enum.(\neg((ap V0f \\ V2n) = V1l)))) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Enum_2Enum) \\ V0f) V1l) c_2Ereal_topology_2Esequentially)))) \Rightarrow (\neg(p (ap (c_2Epred_set_2EFINITE \\ ty_2Erealax_2Ereal) (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal \\ ty_2Erealax_2Ereal) (\lambda V3y \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C \\ ty_2Erealax_2Ereal 2) V3y) (ap (c_2Ebool_2E_3F ty_2Enum_2Enum) \\ (\lambda V4n \in ty_2Enum_2Enum.(ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) \\ V3y) (ap V0f V4n)))))))))))))) \end{aligned}$$