

thm_2Ereal__topology_2ESEQUENCE__UNIQUE__LIMPT (TMMtpLrjXLboyEVBSx2bLnX2gZ93e5ZgAvC)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \quad (2)$$

Definition 7 We define $c_2Epair_2E_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2E21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \quad (3)$$

Definition 8 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (4)$$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Enetord\ A_27a \in \left((2^{A_27a})^{A_27a} \right) (ty_2Ereal_topology_2Enet\ A_27a) \quad (5)$$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A)\ \text{of type } \iota \Rightarrow \iota).$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P))))$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(ap\ (c_2Emin_2E_40\ V2t)\ V1t2))))$

Definition 13 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Enet\ A_27a)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (9)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ V0m)$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Eprim_rec_2E_3C\ V0m\ V1n)$

Definition 16 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Earithmetic_2E_3E\ V0m\ V1n)$

Definition 17 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ c_2Earithmetic_2E_3E_3D\ V0m\ V1n)$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net\ A_27a \in \left((ty_2Ereal_topology_2Enet\ A_27a) \right) \left((2^{A_27a})^{A_27a} \right) \quad (10)$$

Definition 18 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Enum_2Enum))$

Definition 19 We define $c_Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (11)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (12)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (13)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (14)$$

Definition 20 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (t$

Let $c_2Erealax_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Definition 21 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 22 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (16)$$

Definition 23 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (17)$$

Definition 24 We define $c_2Ereal_topology_2E2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 25 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2E)$.

Definition 26 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 27 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E2E$

Definition 28 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (29)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2E}pair_{.2E} C A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2E}pair_{.2E} C A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) V1t))))))) \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2E}pair_{.2E}prod A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V1v) (ap (c_{.2E}pred_{.set} _2EGSPEC A_{.27a} A_{.27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2E}pair_{.2E} C A_{.27a} 2) V1v) c_{.2E}bool_{.2E} ET) = (ap V0f V2x)))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V0x) (c_{.2E}pred_{.set} _2EUNIV A_{.27a})))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0y \in A_{.27b}.(\forall V1s \in (2^{A_{.27a}}).(\forall V2f \in (A_{.27b}^{A_{.27a}}).(((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27b}) V0y) (ap (ap (c_{.2E}pred_{.set} _2EIMAGE A_{.27a} A_{.27b}) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_{.27a}.((V0y = (ap V2f V3x)) \wedge (p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V3x) V1s))))))) \quad (35)$$

Assume the following.

$$\neg(p (ap (c_2Ereal_topology_2Etrivial_limit ty_2Enum_2Enum) c_2Ereal_topology_2Esequentially))) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\ A_27a).(\forall V1f \in (ty_2Erealax_2Ereal^{A_27a}).(\forall V2l \in \\ ty_2Erealax_2Ereal.(\forall V3l.27 \in ty_2Erealax_2Ereal.((\\ \neg(p (ap (c_2Ereal_topology_2Etrivial_limit A_27a) V0net)))) \wedge \\ ((p (ap (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E A_27a) V1f) V2l) \\ V0net)) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E A_27a) \\ V1f) V3l.27) V0net)))))) \Rightarrow (V2l = V3l.27)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} (\forall V0s \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1r \in \\ (ty_2Enum_2Enum^{ty_2Enum_2Enum}).(\forall V2l \in ty_2Erealax_2Ereal. \\ ((\forall V3m \in ty_2Enum_2Enum.(\forall V4n \in ty_2Enum_2Enum. \\ ((p (ap (ap c_2Eprim_rec_2E_3C V3m) V4n)) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\ (ap V1r V3m)) (ap V1r V4n)))))) \wedge (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\ ty_2Enum_2Enum) V0s) V2l) c_2Ereal_topology_2Esequentially))) \Rightarrow \\ (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Enum_2Enum) \\ (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum ty_2Erealax_2Ereal ty_2Enum_2Enum) \\ V0s) V1r)) V2l) c_2Ereal_topology_2Esequentially)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1l \in \\ ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_topology_2Elimit_point_of \\ V1l) (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Erealax_2Ereal) \\ V0f) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))) \Rightarrow (\exists V2r \in \\ (ty_2Enum_2Enum^{ty_2Enum_2Enum}).(\forall V3m \in ty_2Enum_2Enum. \\ (\forall V4n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\ V3m) V4n)) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C (ap V2r V3m)) (ap V2r V4n)))))) \wedge \\ (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Enum_2Enum) \\ (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum ty_2Erealax_2Ereal ty_2Enum_2Enum) \\ V0f) V2r)) V1l) c_2Ereal_topology_2Esequentially)))))) \end{aligned} \quad (39)$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1l \in \\ & \quad ty_2Erealax_2Ereal.(\forall V2l_27 \in ty_2Erealax_2Ereal.((\\ & \quad (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E ty_2Enum_2Enum) \\ V0f) V1l) c_2Ereal_topology_2Esequentially)) \wedge (p (ap (ap c_2Ereal_topology_2Elimit_point_of \\ V2l_27) (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\ (\lambda V3y \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ 2) V3y) (ap (c_2Ebool_2E_3F ty_2Enum_2Enum) (\lambda V4n \in ty_2Enum_2Enum. \\ (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) V3y) (ap V0f V4n)))))))))) \Rightarrow \\ & \quad (V2l_27 = V1l)))))) \end{aligned}$$