

thm_2Ereal__topology_2ESERIES__CMUL
(TMJYB42BUmEb4CSiPHFFKmnz3egAeHjfv2E)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 30 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 31 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E21$

Definition 32 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V$

Definition 33 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Definition 34 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota. (ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (18)$$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 35 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax$

Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet A_27a)}) \quad (20)$$

Definition 36 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota. \lambda V0net \in (ty_2Ereal_topology$

Definition 37 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1net \in (ty_2$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (21)$$

Definition 38 We define $c_2Ereal_topology_2E2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^{A$

Definition 39 We define $c_2Ereal_topology_2ESums$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}). \lambda$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A.27a}). (\forall V1c \in ty_2Erealax_2Ereal. (\forall V2s \in (2^{A.27a}). ((ap\ (ap\ (c.2Eiterate.2ESum\ A.27a)\ V2s)\ (\lambda V3x \in A.27a. (ap\ (ap\ c.2Erealax_2Ereal_mul\ V1c)\ (ap\ V0f\ V3x)))))) = (ap\ (ap\ c.2Erealax_2Ereal_mul\ V1c)\ (ap\ (ap\ c.2Eiterate.2ESum\ A.27a)\ V2s)\ V0f)))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet\ A.27a). (\forall V1f \in (ty_2Erealax_2Ereal^{A.27a}). (\forall V2l \in ty_2Erealax_2Ereal. (\forall V3c \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ (c.2Ereal_topology_2E_2D_2D_3E\ A.27a)\ V1f)\ V2l)\ V0net)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c.2Ereal_topology_2E_2D_2D_3E\ A.27a)\ (\lambda V4x \in A.27a. (ap\ (ap\ c.2Erealax_2Ereal_mul\ V3c)\ (ap\ V1f\ V4x))))))\ (ap\ (ap\ c.2Erealax_2Ereal_mul\ V3c)\ V2l))\ V0net)))))) \quad (30)$$

Theorem 1

$$(\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1x0 \in ty_2Erealax_2Ereal. (\forall V2c \in ty_2Erealax_2Ereal. (\forall V3s \in (2^{ty_2Enum_2Enum}). ((p\ (ap\ (ap\ (ap\ c.2Ereal_topology_2Esums\ V0x)\ V1x0)\ V3s)) \Rightarrow (p\ (ap\ (ap\ (ap\ c.2Ereal_topology_2Esums\ (\lambda V4n \in ty_2Enum_2Enum. (ap\ (ap\ c.2Erealax_2Ereal_mul\ V2c)\ (ap\ V0x\ V4n))))))\ (ap\ (ap\ c.2Erealax_2Ereal_mul\ V2c)\ V1x0))\ V3s)))))) \quad (31)$$