

# thm\_2Ereal\_topology\_2ESERIES\_DIFFS (TMT- TbYZvN4BGHgKZdmGunFVTMf31LqQpRde)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 11** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (6)$$

**Definition 13** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

**Definition 14** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (7)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal)^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 15** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 16** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 18** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (9)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A-27b})^{A-27a}}) \quad (10)$$

**Definition 19** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A-27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A-27b}}) \quad (11)$$

**Definition 20** We define  $c\_2Ereal\_topology\_2Efrom$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2Epred\_set$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{ty\_2Erealax}) \quad (13)$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (16)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 25** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $ty\_2Ereal\_topology\_2Eenet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Eenet\ A0) \quad (18)$$

Let  $c\_2Ereal\_topology\_2Eenetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Eenetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})_{(ty\_2Ereal\_topology\_2Eenet\ A\_27a)}) \quad (19)$$

**Definition 26** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology$

**Definition 27** We define  $c\_Ereal\_topology\_Eeventually$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2Ereal\_topology\_Eeventually\_net A\_27a)$

**Definition 28** We define  $c\_Earithmic\_E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 29** We define  $c\_Earithmic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Ereal\_topology\_Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ereal\_topology\_Emk\_net \\ A\_27a \in ((ty\_2Ereal\_topology\_Enet\ A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (20)$$

**Definition 30** We define  $c\_Ereal\_topology\_Esequentially$  to be  $(ap\ (c\_Ereal\_topology\_Emk\_net\ ty\_2Ereal\_topology\_Esequentially\_net\ A\_27a))$

**Definition 31** We define  $c\_Eiterate\_E\_2E\_2E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_Ebool\_EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 33** We define  $c\_Epred\_set\_EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_Ebool\_EIN\ A\_27a)\ V1t\ V0s)$

**Definition 34** We define  $c\_Eiterate\_Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_Eiterate\_E\_2E\_2E\ A\_27a)\ V0op)$

**Definition 35** We define  $c\_Eiterate\_Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_Eiterate\_Eneutral\ A\_27a)\ V1op\ V0op)$

**Definition 36** We define  $c\_Epred\_set\_EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_Ebool\_EIN\ A\_27a)\ V1s\ V0x)$

**Definition 37** We define  $c\_Epred\_set\_EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_Ebool\_EIN\ A\_27a\ V0x)$

**Definition 38** We define  $c\_Epred\_set\_EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_Ebool\_EIN\ A\_27a)\ V0s)$

**Definition 39** We define  $c\_Eiterate\_EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V1op \in ((A\_27b^{A\_27b})^{A\_27b}).(ap\ (c\_Eiterate\_Eneutral\ A\_27a)\ V1op\ V0f)$

**Definition 40** We define  $c\_Eiterate\_Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V1op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_Eiterate\_Eneutral\ A\_27a)\ V1op\ V0op)$

**Definition 41** We define  $c\_Eiterate\_ESum$  to be  $\lambda A\_27a : \iota.(ap\ (c\_Eiterate\_Eiterate\ A\_27a\ ty\_2Erealax\_2Ereal)\ V0op)$

Let  $c\_Ereal\_topology\_EDist : \iota$  be given. Assume the following.

$$c\_Ereal\_topology\_EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (21)$$

Let  $c\_Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (22)$$

**Definition 42** We define  $c\_Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 43** We define  $c\_Ereal\_topology\_E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A\_27a})$

**Definition 44** We define  $c\_Ereal\_topology\_ESums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1op \in (ty\_2Enum\_2Enum)$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (26) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p \ V0t)))))) \quad (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\ & V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF \\ & V0t1) \ V1t2) = V1t2)))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\ & (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))))) \quad (32) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a\ \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
& V5y\_27)))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1m \in \\
& ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ((ap\ (ap\ (c\_2Eiterate\_2ESum \\
& ty\_2Enum\_2Enum)\ (ap\ (ap\ c\_2Eiterate\_2E\_2E\_2E\ V1m)\ V2n))\ (\lambda V3k \in \\
& ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ V0f\ V3k))\ (ap \\
& V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V3k)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) = (ap \\
& (ap\ (ap\ (c\_2Ebool\_2ECOND\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\
& V1m)\ V2n))\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ V0f\ V1m))\ (ap\ V0f\ (ap \\
& (ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) = V0x)) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (2^{ty\_2Enum\_2Enum}). ((p\ (ap\ (ap\ (c\_2Ereal\_topology\_2Eeventually \\
& ty\_2Enum\_2Enum)\ V0p)\ c\_2Ereal\_topology\_2Esequentially)) \Leftrightarrow \\
& (\exists V1N \in ty\_2Enum\_2Enum. (\forall V2n \in ty\_2Enum\_2Enum. ( \\
& (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1N)\ V2n)) \Rightarrow (p\ (ap\ V0p\ V2n))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0k \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ ty\_2Enum\_2Enum)\ (ap\ c\_2Ereal\_topology\_2Efrom \\
& V0k))\ (ap\ (ap\ c\_2Eiterate\_2E\_2E\_2E\ c\_2Enum\_2E0)\ V1n)) = (ap\ (ap \\
& c\_2Eiterate\_2E\_2E\_2E\ V0k)\ V1n)))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0net \in (ty\_2Ereal\_topology\_2Enet \\
& A\_27a). (\forall V1a \in ty\_2Erealax\_2Ereal. (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& A\_27a)\ (\lambda V2x \in A\_27a.V1a))\ V1a)\ V0net))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0net \in (ty\_2Ereal\_topology\_2Enet \\
& \quad A.27a).(\forall V1f \in (ty\_2Erealax\_2Ereal^{A-27a}).(\forall V2g \in \\
& \quad (ty\_2Erealax\_2Ereal^{A-27a}).(\forall V3l \in ty\_2Erealax\_2Ereal. \\
& (\forall V4m \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad A.27a)\ V1f)\ V3l)\ V0net))) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad A.27a)\ V2g)\ V4m)\ V0net)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad A.27a)\ (\lambda V5x \in A.27a.(ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ V1f\ V5x)) \\
& \quad (ap\ V2g\ V5x))))\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V3l\ V4m))\ V0net))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0net \in (ty\_2Ereal\_topology\_2Enet \\
& \quad A.27a).(\forall V1f \in (ty\_2Erealax\_2Ereal^{A-27a}).(\forall V2g \in \\
& \quad (ty\_2Erealax\_2Ereal^{A-27a}).(\forall V3l \in ty\_2Erealax\_2Ereal. \\
& \quad (((p\ (ap\ (ap\ (c\_2Ereal\_topology\_2Eeventually\ A.27a)\ (\lambda V4x \in \\
& \quad A.27a.(ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Erealax\_2Ereal)\ (ap\ V1f\ V4x)) \\
& \quad (ap\ V2g\ V4x))))\ V0net))) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad A.27a)\ V1f)\ V3l)\ V0net)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad A.27a)\ V2g)\ V3l)\ V0net))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1l \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap \\
& \quad (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E\ ty\_2Enum\_2Enum)\ V0f)\ V1l) \\
& \quad c\_2Ereal\_topology\_2Esequentially)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad ty\_2Enum\_2Enum)\ (\lambda V3i \in ty\_2Enum\_2Enum.(ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V3i)\ V2k))))\ V1l)\ c\_2Ereal\_topology\_2Esequentially)))))) \\
& \hspace{15em} (41)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1k \in \\
& \quad ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (ap\ (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad ty\_2Enum\_2Enum)\ V0f)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \\
& \quad c\_2Ereal\_topology\_2Esequentially)) \Rightarrow (p\ (ap\ (ap\ (ap\ c\_2Ereal\_topology\_2Esums \\
& \quad (\lambda V2n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ V0f \\
& \quad V2n))\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap \\
& \quad V0f\ V1k))\ (ap\ c\_2Ereal\_topology\_2Efrom\ V1k))))))
\end{aligned}$$