

# thm\_2Ereal\_\_topology\_2ESERIES\_\_FINITE\_\_SUPPORT (TMYW2bxWqSSTmjnx2cK3VKZCNKyhmCKFb1)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$   
of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 10** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A.27a : \iota. \lambda V0x \in A.27a. \lambda V1s \in (2^{A-27a}). (ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Ebool\_2E21 2$  to be  $(ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A.27a : \iota. (\lambda V0x \in A.27a. c\_2Ebool\_2E21 2)$ .

**Definition 13** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A.27a : \iota. \lambda V0s \in (2^{A-27a}). (ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (5)$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \quad (6)$$

**Definition 14** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2. (ap (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

Let  $c\_2Eenum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EREP\_num \in (\omega^{ty\_2Eenum\_2Eenum}) \quad (7)$$

Let  $c\_2Eenum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

Let  $c\_2Eenum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EABS\_num \in (ty\_2Eenum\_2Eenum^{\omega}) \quad (9)$$

**Definition 15** We define  $c\_2Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum. (ap\ c\_2Eenum\_2EABS\_num\ m)$ .

**Definition 16** We define  $c\_2Emin\_2E40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 17** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A.27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E40\ P)\ V0t)))$ .

**Definition 18** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum. \lambda V1n \in ty\_2Eenum\_2Eenum. (ap\ c\_2Eprim\_rec\_2E3C\ m\ n)$ .

**Definition 19** We define  $c\_2Earithmetic\_2E3C\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum. \lambda V1n \in ty\_2Eenum\_2Eenum. (ap\ c\_2Earithmetic\_2E3C\_3D\ m\ n)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (10)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (11)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (12)$$

**Definition 20** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap (c\_Emin\_E40 (t$   
Let  $c\_Erealax\_Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_lt \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal)) \quad (13)$$

**Definition 21** We define  $c\_Erealax\_Ereal\_lt$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 22** We define  $c\_Earithmic\_E\_3E$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 23** We define  $c\_Earithmic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

Let  $ty\_Ereal\_topology\_Eenet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_Ereal\_topology\_Eenet A0) \quad (14)$$

Let  $c\_Ereal\_topology\_Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow c\_Ereal\_topology\_Emk\_net \\ A.27a \in ((ty\_Ereal\_topology\_Eenet A.27a)^{(2^{A-27a})^{A-27a}}) \end{aligned} \quad (15)$$

**Definition 24** We define  $c\_Ereal\_topology\_Esequentially$  to be  $(ap (c\_Ereal\_topology\_Emk\_net ty\_Ereal\_topology\_Eenet))$

Let  $c\_Eenum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_EZERO\_REP \in omega \quad (16)$$

**Definition 25** We define  $c\_Eenum\_E0$  to be  $(ap c\_Eenum\_EABS\_num c\_Eenum\_EZERO\_REP)$ .

**Definition 26** We define  $c\_Eiterate\_E\_2E\_2E$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 27** We define  $c\_Epred\_set\_EINTER$  to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c\_Epred\_set\_EINTER))$

Let  $c\_Erealax\_Etrealm\_add : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_add \in (((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal))^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)}) \quad (17)$$

Let  $c\_Erealax\_Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_eq \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)) \quad (18)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal)^{(2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})} \quad (19)$$

**Definition 28** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)$

**Definition 29** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal$

**Definition 30** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap (c\_2Emin$

**Definition 31** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 32** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 33** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

**Definition 34** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 35** We define  $c\_2Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eiterate\_2Eiterate A\_27a ty\_2Erealax$

Let  $c\_2Ereal\_topology\_2Eenetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ereal\_topology\_2Eenetord A\_27a \in \left( \left( (2^{A\_27a})^{A\_27a} \right)^{(ty\_2Ereal\_topology\_2Eenet A\_27a)} \right) \quad (20)$$

**Definition 36** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology$

**Definition 37** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2$

**Definition 38** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^A$

**Definition 39** We define  $c\_2Ereal\_topology\_2ESums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))) \Rightarrow (p ( \\ & ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \end{aligned} \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\
& (p \ V1B) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \quad (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) \\
V2p) (ap (ap c\_2Eiterate\_2E\_2E\_2E V0m) V1n))) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V0m) V2p)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) V1n))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}). \\
& \quad (\forall V1u \in (2^{A\_27a}). (\forall V2v \in (2^{A\_27a}). (((p (ap (ap ( \\
& \quad \quad c\_2Epred\_set\_2ESUBSET A\_27a) V1u) V2v)) \wedge (\forall V3x \in A\_27a. \\
& \quad ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V2v)) \wedge (\neg (p (ap (ap (c\_2Ebool\_2EIN \\
& \quad A\_27a) V3x) V1u)))) \Rightarrow ((ap V0f V3x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)))) \Rightarrow ((ap (ap (c\_2Eiterate\_2ESum A\_27a) V2v) V0f) = \\
& \quad (ap (ap (c\_2Eiterate\_2ESum A\_27a) V1u) V0f)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\
& \quad (2^{A\_27a}). (\forall V2x \in A\_27a. ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) \\
V2x) (ap (ap (c\_2Epred\_set\_2EINTER A\_27a) V0s) V1t))) \Leftrightarrow ((p (ap (37) \\
& \quad (ap (c\_2Ebool\_2EIN A\_27a) V2x) V0s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN \\
& \quad A\_27a) V2x) V1t)))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}). \\
& \quad (\forall V1s \in (2^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) \\
V1s)) \Rightarrow (\exists V2a \in ty\_2Enum\_2Enum. (\forall V3x \in A\_27a. ((p ( \\
& \quad ap (ap (c\_2Ebool\_2EIN A\_27a) V3x) V1s)) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad (ap V0f V3x)) V2a))))))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Ereal\_topology\_2EDist \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (39) \\
& \quad V0x) V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1l \in \\
& \quad ty\_2Erealax\_2Ereal. ((p (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& \quad ty\_2Enum\_2Enum) V0s) V1l) c\_2Ereal\_topology\_2Esequentially)) \Leftrightarrow \\
& \quad (\forall V2e \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) V2e)) \Rightarrow (\exists V3N \in \\
& \quad ty\_2Enum\_2Enum. (\forall V4n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V3N) V4n)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (ap V0s V4n)) V1l)) V2e))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (52)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1s \in \\ & \quad (2^{ty\_2Enum\_2Enum}). (\forall V2k \in (2^{ty\_2Enum\_2Enum}). (((p ( \\ ap (c\_2Epred\_set\_2EFINITE ty\_2Enum\_2Enum) (ap (ap (c\_2Epred\_set\_2EINTER \\ & \quad ty\_2Enum\_2Enum) V1s) V2k))) \wedge (\forall V3x \in ty\_2Enum\_2Enum. (( \\ & \quad (p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V3x) V2k)) \wedge (\neg(p (ap ( \\ & \quad ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V3x) V1s)))) \Rightarrow ((ap V0f V3x) = \\ & \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))) \Rightarrow (p (ap (ap (ap \\ & \quad c\_2Ereal\_topology\_2Esums V0f) (ap (ap (c\_2Eiterate\_2ESum ty\_2Enum\_2Enum) \\ & \quad (ap (ap (c\_2Epred\_set\_2EINTER ty\_2Enum\_2Enum) V1s) V2k)) V0f)) \\ & \quad V2k)))))) \end{aligned}$$