

thm_2Ereal__topology_2ESERIES__INJECTIVE__IMAGE (TMTZ4yRHicvrrf6QFJeoEt3GiEpGuN3RPop)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Ereal_topology_2Eenet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ereal_topology_2Eenet A0) \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (2)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (3)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ & A_27a \in ((ty_2Ereal_topology_2Enet\ A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 15 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Enum_2EZERO_REP).$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Definition 17 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (9)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (10)$$

Definition 19 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 20 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 21 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2EIN\ s)\ t)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (11)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (12)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (13)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E40\ a)\ (c_2Erealax_2Ereal_REP_CLASS\ a))$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (16)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal). (c_2Erealax_2Ereal_ABS_CLASS\ r)$

Definition 24 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (c_2Erealax_2Ereal_add\ T1\ T2)$

Definition 25 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota. \lambda V0op \in ((A_27a^{A_27a})^{A_27a}). (ap\ (c_2Emin_2E40\ op)\ (c_2Erealax_2Ereal_add\ op\ op))$

Definition 26 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V1x \in (A_27b^{A_27b}). (c_2Eiterate_2Eneutral\ op\ x)$

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2EIF\ t1\ t2\ t1))))$

Definition 28 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2ECOND\ s)\ x)$

Definition 29 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EIF\ x\ (c_2Ebool_2ECOND\ (c_2Ebool_2ECOND\ s)\ x)))$

Definition 30 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2ECOND\ s)\ (c_2Ebool_2ECOND\ s))$

Definition 31 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in ((A_27a^{A_27a})^{A_27b}). \lambda V1x \in (A_27b^{A_27b}). (c_2Eiterate_2Esupport\ f\ x)$

Definition 32 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Definition 33 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota. (ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (17)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (18)$$

Definition 34 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in ((2^{A_27a})^{A_27a})(ty_2Ereal_topology_2Eenet A_27a) \quad (19)$$

Definition 35 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota. \lambda V0net \in (ty_2Ereal_topology_2Etrivial_limit$

Definition 36 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1net \in (ty_2Ereal_topology_2Eeventually$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (20)$$

Definition 37 We define $c_2Ereal_topology_2E2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^{A_27a})$

Definition 38 We define $c_2Ereal_topology_2ESums$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}). \lambda V1f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})$

Definition 39 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1f \in (A_27c^{A_27a})$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (21)$$

Definition 40 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap c_2Erealax_2Ereal_neg$

Definition 41 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal$

Definition 42 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal$

Definition 43 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal. (ap (ap (ap (c_2Ebool_2ECONJ$

Definition 44 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 45 We define $c_2Ereal_topology_2Esummable$ to be $\lambda V0s \in (2^{ty_2Eenum_2Eenum}). \lambda V1f \in (ty_2Ereal_topology_2Esummable$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0net \in (ty_2Ereal_topology_2Enet \\ & A_27a).(\forall V1f \in (ty_2Erealax_2Ereal^{A_27a}).(\forall V2g \in \\ & (ty_2Erealax_2Ereal^{A_27a}).(\forall V3l \in ty_2Erealax_2Ereal. \\ & ((p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E\ A_27a)\ (\lambda V4x \in \\ & A_27a.(ap\ (ap\ c_2Ereal_2Ereal_sub\ (ap\ V1f\ V4x))\ (ap\ V2g\ V4x)))))) \\ & (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V0net)) \Rightarrow ((p\ (ap\ (\\ & ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E\ A_27a)\ V1f)\ V3l)\ V0net)) \Leftrightarrow \\ & (p\ (ap\ (ap\ (ap\ (c_2Ereal_topology_2E_2D_2D_3E\ A_27a)\ V2g)\ V3l)\ \\ & V0net)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1s \in \\
& (2^{ty_2Enum_2Enum}).(\forall V2f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). \\
& (((p (ap (ap (ap c_2Ereal_topology_2Esummable (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Enum_2Enum ty_2Enum_2Enum) V2f) V1s)) (\lambda V3n \in ty_2Enum_2Enum. \\
& (ap c_2Ereal_2Eabs (ap V0x V3n)))))) \wedge (\forall V4m \in ty_2Enum_2Enum. \\
& (\forall V5n \in ty_2Enum_2Enum. (((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) \\
& V4m) V1s)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V5n) V1s)) \wedge \\
& ((ap V2f V4m) = (ap V2f V5n)))) \Rightarrow (V4m = V5n)))) \Rightarrow (p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3D \\
& ty_2Enum_2Enum) (\lambda V6n \in ty_2Enum_2Enum. (ap (ap c_2Ereal_2Ereal_sub \\
& (ap (ap (c_2Eiterate_2ESum ty_2Enum_2Enum) (ap (ap (c_2Epred_set_2EINTER \\
& ty_2Enum_2Enum) (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2f) V1s)) (ap (ap c_2Eiterate_2E_2E_2E c_2Enum_2E0) \\
& V6n))) V0x)) (ap (ap (c_2Eiterate_2ESum ty_2Enum_2Enum) (ap (ap \\
& (c_2Epred_set_2EINTER ty_2Enum_2Enum) V1s) (ap (ap c_2Eiterate_2E_2E_2E \\
& c_2Enum_2E0) V6n))) (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum ty_2Erealax_2Ereal \\
& ty_2Enum_2Enum) V0x) V2f)))))) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& c_2Ereal_topology_2Esequentially)))))) \quad (28)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1s \in \\
& (2^{ty_2Enum_2Enum}).(\forall V2f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). \\
& (\forall V3l \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_topology_2Esummable \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Enum_2Enum) \\
& V2f) V1s)) (\lambda V4n \in ty_2Enum_2Enum. (ap c_2Ereal_2Eabs (ap V0x \\
& V4n)))))) \wedge (\forall V5m \in ty_2Enum_2Enum. (\forall V6n \in ty_2Enum_2Enum. \\
& (((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V5m) V1s)) \wedge ((p (ap \\
& (ap c_2Ebool_2EIN ty_2Enum_2Enum) V6n) V1s)) \wedge ((ap V2f V5m) = (\\
& ap V2f V6n)))) \Rightarrow (V5m = V6n)))) \Rightarrow ((p (ap (ap (ap c_2Ereal_topology_2ESums \\
& (ap (ap (c_2Ecombin_2Eo ty_2Enum_2Enum ty_2Erealax_2Ereal ty_2Enum_2Enum) \\
& V0x) V2f)) V3l) V1s)) \Leftrightarrow (p (ap (ap (ap c_2Ereal_topology_2ESums \\
& V0x) V3l) (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Enum_2Enum) \\
& V2f) V1s)))))))))
\end{aligned}$$