

thm_2Ereal__topology_2ESERIES__RESTRICT
(TMYqpcESTon-
phzRuXP3bvGDkhjTbpF9GQ5U)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{3}$$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda V1f \in V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(\lambda V0P \in (c_2Emin_2E_3D (2^{A_27a}))))$

Definition 6 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(\lambda V1t \in V0s)$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota.$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (7)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A. 27a. nonempty\ A. 27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A. 27a \in ((ty_2Ereal_topology_2Enet\ A. 27a)^{(2^{A-27a})^{A-27a}}) \end{aligned} \quad (8)$$

Definition 18 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Ereal_topology_2Esequentially$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Definition 20 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. nonempty\ A. 27a \Rightarrow \forall A. 27b. nonempty\ A. 27b \Rightarrow c_2Epair_2EABS_prod\ A. 27a\ A. 27b \in ((ty_2Epair_2Eprod\ A. 27a\ A. 27b)^{(2^{A-27b})^{A-27a}}) \quad (11)$$

Definition 21 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(12)

Definition 22 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Erealax_2Ereal_REP_CLASS : \iota)$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(13)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS})$$
(14)

Definition 24 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{ty_2Erealax_2Ereal_REP})^{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal})^{ty_2Erealax_2Etrealm_add}$$
(15)

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal})^{ty_2Erealax_2Etrealm_eq}$$
(16)

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_ABS_CLASS}}$$
(17)

Definition 25 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 26 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 27 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota. \lambda V0op \in ((A_27a^{A_27a})^{A_27a}). (ap (c_2Emin$

Definition 28 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0op \in ((A_27b^{A_27b})^{A_27b}). \lambda V$

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 30 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2E$

Definition 31 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF).$

Definition 32 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Definition 33 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 34 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 35 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Erealax$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (18)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 36 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Ereal_topology_2Eenetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Eenetord A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Eenet A_27a)} \quad (20)$$

Definition 37 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota.\lambda V0net \in (ty_2Ereal_topology$

Definition 38 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2$

Definition 39 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^{A$

Definition 40 We define $c_2Ereal_topology_2Esums$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).\lambda$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{28}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \tag{29}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{30}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\
& V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). \\ & \quad (\forall V3x \in A_27a. ((ap (ap (ap (ap (c_2Ebool_2ECOND (A_27b^{A_27a}) \\ & \quad V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V0b) (ap \\ & \quad V1f V3x)) (ap V2g V3x)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & \quad (\forall V3y \in A_27a. ((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & \quad V1b) V2x) V3y)) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f \\ & \quad V2x)) (ap V0f V3y)))))) \end{aligned} \quad (38)$$

Assume the following.

$$2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & \quad (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & \quad (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & \quad ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & \quad V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & \quad V5y_27)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). \\ & \quad (\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). \\ & \quad ((\forall V3x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V2s)) \Rightarrow \\ & \quad ((ap V0f V3x) = (ap V1g V3x)))) \Rightarrow ((ap (ap (c_2Eiterate_2ESum A_27a) \\ & \quad V2s) V0f) = (ap (ap (c_2Eiterate_2ESum A_27a) V2s) V1g)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). \\
& \quad (\forall V1u \in (2^{A_27a}). (\forall V2v \in (2^{A_27a}). (((p\ (ap\ (ap\ (\\
& \quad \quad c_2Epred_set_2ESUBSET\ A_27a\ V1u\ V2v))) \wedge (\forall V3x \in A_27a. \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a\ V3x\ V2v))) \wedge (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad \quad A_27a\ V3x\ V1u)))))) \Rightarrow ((ap\ V0f\ V3x) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)))))) \Rightarrow ((ap\ (ap\ (c_2Eiterate_2ESum\ A_27a\ V2v\ V0f) = \\
& \quad (ap\ (ap\ (c_2Eiterate_2ESum\ A_27a\ V1u\ V0f))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V2x) (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a\ V0s\ V1t)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad \quad A_27a\ V2x\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad \quad \quad A_27a\ V2x\ V1t))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}). ((ap\ (\\
& \quad ap\ (c_2Epred_set_2EINTER\ A_27a) (c_2Epred_set_2EUNIV\ A_27a)) \\
& \quad V0s) = V0s)) \wedge (\forall V1s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A_27a\ V1s) (c_2Epred_set_2EUNIV\ A_27a)) = V1s)))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False)) \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee (\neg(p\ V2r)) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (ap (ap (ap (c_2Ebool.2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\
& (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V3s)))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(\\
& p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{60}$$

Theorem 1

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1k \in \\ & (2^{ty_2Enum_2Enum}).(\forall V2l \in ty_2Erealax_2Ereal.((p (ap \\ & (ap (ap c_2Ereal_topology_2Esums (\lambda V3n \in ty_2Enum_2Enum. \\ & (ap (ap (ap (c_2Ebool_2ECOND ty_2Erealax_2Ereal) (ap (ap (c_2Ebool_2EIN \\ & ty_2Enum_2Enum) V3n) V1k)) (ap V0f V3n)) (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)))) V2l) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))) \Leftrightarrow \\ & (p (ap (ap (ap c_2Ereal_topology_2Esums V0f) V2l) V1k)))) \end{aligned}$$