

thm\_2Ereal\_\_topology\_2ESERIES\_\_SUM  
(TMN1G8aHVZpcQBPV476KXA3SVLhXB9vcijx)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

**Definition 5** We define  $c\_2Ecombin\_2E\_2K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** We define  $c\_2Ecombin\_2E\_2S$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 7** We define  $c\_2Ecombin\_2E\_2I$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2S A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 8** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P)))$

**Definition 11** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (ap V1t V0s)))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (4)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E.40\ t$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (5)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 15** We define  $c\_2Ebool\_2E.2F.5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V2t \in 2$

**Definition 16** We define  $c\_2Eiterate\_2Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_2Emin\_2E.40\ t$

**Definition 17** We define  $c\_2Ebool\_2E.F$  to be  $(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 18** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E.3D.3D.3E\ V0t)\ c\_2Ebool\_2E.7E))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (8)$$

**Definition 19** We define  $c\_2Epair\_2E.2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Emin\_2E.40\ t$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (9)$$

**Definition 20** We define  $c\_2Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}). \lambda V$

**Definition 21** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 22** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in$

**Definition 23** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2$

**Definition 24** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF).$

**Definition 25** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E\_21) 2)$

**Definition 26** We define  $c\_2Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}). \lambda V$

**Definition 27** We define  $c\_2Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}). \lambda V$

**Definition 28** We define  $c\_2Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eiterate\_2Eiterate A\_27a ty\_2Erealax$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (12)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (13)$$

**Definition 29** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num$

**Definition 30** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 31** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Enet\ A0) \quad (14)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} &\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ &A\_27a \in ((ty\_2Ereal\_topology\_2Enet\ A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (15)$$

**Definition 33** We define  $c\_Ereal\_topology\_Esequentially$  to be  $(ap (c\_Ereal\_topology\_Emk\_net ty\_2E))$ .  
Let  $c\_Enum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_EZERO\_REP \in \omega \tag{16}$$

**Definition 34** We define  $c\_Enum\_E0$  to be  $(ap c\_Enum\_EABS\_num c\_Enum\_EZERO\_REP)$ .

**Definition 35** We define  $c\_Earithmic\_E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 36** We define  $c\_Eiterate\_E\_2E\_2E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 37** We define  $c\_Epred\_set\_EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_E))$

Let  $c\_Ereal\_topology\_EDist : \iota$  be given. Assume the following.

$$c\_Ereal\_topology\_EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)}) \tag{17}$$

Let  $c\_Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)) \tag{18}$$

**Definition 38** We define  $c\_Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_Ereal\_topology\_Eenetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_Ereal\_topology\_Eenetord A\_27a \in ((2^{A\_27a})^{A\_27a})(ty\_2Ereal\_topology\_Eenet A\_27a) \tag{19}$$

**Definition 39** We define  $c\_Ereal\_topology\_Etrivial\_limit$  to be  $\lambda A\_27a : \iota. \lambda V0net \in (ty\_2Ereal\_topology\_Eenet A\_27a).$

**Definition 40** We define  $c\_Ereal\_topology\_Eeventually$  to be  $\lambda A\_27a : \iota. \lambda V0p \in (2^{A\_27a}). \lambda V1net \in (ty\_2Ereal\_topology\_Eenet A\_27a).$

Let  $c\_Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{20}$$

**Definition 41** We define  $c\_Ereal\_topology\_E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Erealax\_2Ereal^{A\_27a}).$

**Definition 42** We define  $c\_Ereal\_topology\_Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (31)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\neg(\forall V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in 2. (((\exists V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in A\_27a. ((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). (((p\ V0P) \vee (\exists V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (41)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (42)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0P \in ((2^{A.27b})^{A.27a}).(\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ \forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((ap\ (c.2Ecombin.2EI\ A.27a)\ V0x) = V0x)) \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ \\ V2x)\ (ap\ (ap\ (c.2Epred\_set.2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} (\forall V0f \in (ty.2Erealax.2Erealty.2Enum.2Enum).(\forall V1s \in \\ (2^{ty.2Enum.2Enum}).(\forall V2k \in (2^{ty.2Enum.2Enum}).(((p\ ( \\ ap\ (c.2Epred\_set.2EFINITE\ ty.2Enum.2Enum)\ (ap\ (ap\ (c.2Epred\_set.2EINTER \\ ty.2Enum.2Enum)\ V1s)\ V2k))) \wedge (\forall V3x \in ty.2Enum.2Enum.(( \\ (p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Enum.2Enum)\ V3x)\ V2k)) \wedge (\neg(p\ (ap\ ( \\ ap\ (c.2Ebool.2EIN\ ty.2Enum.2Enum)\ V3x)\ V1s)))) \Rightarrow ((ap\ V0f\ V3x) = \\ (ap\ c.2Ereal.2Ereal\_of\_num\ c.2Enum.2E0)))) \Rightarrow (p\ (ap\ (ap\ (ap \\ c.2Ereal\_topology.2Esums\ V0f)\ (ap\ (ap\ (c.2Eiterate.2ESum\ ty.2Enum.2Enum) \\ (ap\ (ap\ (c.2Epred\_set.2EINTER\ ty.2Enum.2Enum)\ V1s)\ V2k))\ V0f)) \\ V2k)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (61)$$



Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (63)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1l \in \\ & \quad ty\_2Erealax\_2Ereal.(\forall V2k \in (2^{ty\_2Enum\_2Enum}).(\forall V3s \in \\ & (2^{ty\_2Enum\_2Enum}).((p (ap (c\_2Epred\_set\_2EFINITE ty\_2Enum\_2Enum) \\ & V3s)) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Enum\_2Enum) V3s) \\ & V2k)) \wedge ((\forall V4x \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (c\_2Ebool\_2EIN \\ & ty\_2Enum\_2Enum) V4x) V3s))) \Rightarrow ((ap V0f V4x) = (ap c\_2Ereal\_2Ereal\_of\_num \\ & \quad c\_2Enum\_2E0)))) \wedge ((ap (ap (c\_2Eiterate\_2ESum ty\_2Enum\_2Enum) \\ & V3s) V0f) = V1l)))) \Rightarrow (p (ap (ap (ap c\_2Ereal\_topology\_2Esums V0f) \\ & \quad V1l) V2k)))))) \end{aligned}$$