

thm\_2Ereal\_\_topology\_2ESETDIST\_\_EQ\_\_0\_\_CLOSED\_\_COMPACT  
(TMLiPhYUQzouhB6DhvZG2dq1PjeX1d3sRuF)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}). (ap V0P (ap (c_2Emin_2E_40 A_{27a} P))))$

**Definition 5** We define `c_2Ebool_2EIN` to be  $\lambda A_{27a} : \iota.(\lambda V0x \in A_{27a}.(\lambda V1f \in (2^{A_{27a}}). (ap V1f V0x)))$

**Definition 6** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 7** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}). (ap (ap (c_2Emin_2E_3D (2^{A_{27a}})) P) V0P)))$

**Definition 8** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c_2Epair_2EABS\_prod A_{27a} A_{27b} \in ((ty_2Epair_2Eprod A_{27a} A_{27b})^{(2^{A_{27b}})^{A_{27a}}}) \quad (2)$$

**Definition 9** We define `c_2Epair_2E_2C` to be  $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda V0x \in A_{27a}.\lambda V1y \in A_{27b}.(ap (c_2Epair_2E_2C A_{27a} A_{27b}) (V0x V1y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}})$$
(3)

**Definition 10** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega$$
(4)

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum$$
(5)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega})$$
(6)

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal$$
(7)

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})$$
(8)

Let  $c\_2Ereal\_topology\_2Esetdist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2Esetdist \in (ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod\ (2^{ty\_2Erealx\_2Ereal})\ (2^{ty\_2Erealx\_2Ereal}))})$$
(9)

**Definition 14** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2ET)$ .

**Definition 15** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E21))$ .

**Definition 16** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E7E\ c\_2Epred\_set\_2EEMPTY))$ .

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal\ ty\_2Erealx\_2Ereal)})$$
(10)

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal$$
(11)

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal})$$
(12)

**Definition 17** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (t$   
Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)) \quad (13)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 19** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E2$

**Definition 20** We define  $c\_2Ereal\_topology\_2EClosed$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap c\_2Ereal\_topo$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (15)$$

**Definition 21** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 22** We define  $c\_2Eprim\_rec\_2E3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 23** We define  $c\_2Earithmetic\_2E3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 24** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (\lambda V2t \in$

**Definition 25** We define  $c\_2Earithmetic\_2E3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ereal\_topology\_2Enet A0) \quad (16)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ A.27a \in ((ty\_2Ereal\_topology\_2Enet A.27a)^{(2^{A.27a})^{A.27a}}) \end{aligned} \quad (17)$$

**Definition 26** We define  $c\_2Ereal\_topology\_2Esequentially$  to be  $(ap (c\_2Ereal\_topology\_2Emk\_net ty\_2E$

**Definition 27** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A.27c}).\lambda V1$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c\_2Ereal\_topology\_2Enetord A.27a \in ((2^{A.27a})^{A.27a})(ty\_2Ereal\_topology\_2Enet A.27a) \quad (18)$$

**Definition 28** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A.27a : \iota.\lambda V0net \in (ty\_2Ereal\_topology$

**Definition 29** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2$

**Definition 30** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^A$

**Definition 31** We define  $c\_2Ereal\_topology\_2Ecompact$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (c\_2Ebool\_2E$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \quad (24) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \quad (26) \end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\forall V3x \in A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. ((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3)))))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)) \quad (30)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (34)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). ((ap\ c\_2Ereal\_topology\_2Esetdist\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{ty\_2Erealax\_2Ereal})\ V0s)\ V1t)) = (ap\ c\_2Ereal\_topology\_2Esetdist\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal})\ V1t)\ V0s)))))) \quad (35)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& (((p (ap\ c\_2Ereal\_topology\_2Ecompact\ V0s)) \wedge (p (ap\ c\_2Ereal\_topology\_2EClosed \\
& \quad V1t))) \Rightarrow (((ap\ c\_2Ereal\_topology\_2Esetdist\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t)) = \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \Leftrightarrow ((V0s = (c\_2Epred\_set\_2EEMPTY \\
& \quad ty\_2Erealax\_2Ereal)) \vee ((V1t = (c\_2Epred\_set\_2EEMPTY\ ty\_2Erealax\_2Ereal)) \vee \\
& \quad (\neg((ap\ (ap\ (c\_2Epred\_set\_2EINTER\ ty\_2Erealax\_2Ereal)\ V0s)\ V1t) = \\
& \quad (c\_2Epred\_set\_2EEMPTY\ ty\_2Erealax\_2Ereal))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (47)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\ & (((p (ap\ c\_2Ereal\_topology\_2EClosed\ V0s)) \wedge (p (ap\ c\_2Ereal\_topology\_2Ecompact \\ & V1t)))) \Rightarrow (((ap\ c\_2Ereal\_topology\_2Esetdist\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{ty\_2Erealax\_2Ereal})\ (2^{ty\_2Erealax\_2Ereal}))\ V0s)\ V1t)) = \\ & (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \Leftrightarrow ((V0s = (c\_2Epred\_set\_2EEMPTY \\ & ty\_2Erealax\_2Ereal)) \vee ((V1t = (c\_2Epred\_set\_2EEMPTY\ ty\_2Erealax\_2Ereal)) \vee \\ & (\neg((ap\ (ap\ (c\_2Epred\_set\_2EINTER\ ty\_2Erealax\_2Ereal)\ V0s)\ V1t) = \\ & (c\_2Epred\_set\_2EEMPTY\ ty\_2Erealax\_2Ereal)))))))))) \end{aligned}$$