

thm_2Ereal__topology_2ESETDIST__FRONTIERS (TMUsNrdcgXo8wmtgAeYUpiRpipK9gTtM4fQ)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40 (2^{A_27a}))$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Ereal_topology_2Eopen_segment : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eopen_segment \in ((2^{ty_2Erealx_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)}) \tag{3}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (4)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (5)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))\ c_2Enum_2E0)$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (13)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (14)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (15)$$

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ t$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Definition 15 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 17 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Elist_2EHd : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHd\ A_27a \in (A_27a^{(ty_2Elist_2Elist\ A_27a)}) \quad (17)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (19)$$

Let $c_2Erealax_2Etrealmul_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (20)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (21)$$

Definition 18 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 19 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (22)$$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (23)$$

Definition 20 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (24)$$

Definition 21 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 22 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b} A_27a)}) \end{aligned} \quad (25)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (26)$$

Definition 24 We define $c_2Ereal_topology_2Eclosed_segment$ to be $\lambda V0l \in (ty_2Elist_2Elist\ (ty_2Epair$

Definition 25 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 26 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 27 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 28 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 29 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x))))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x))))))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (46)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x))))))) \quad (47)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (53)$$

Assume the following.

$$2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (54)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in A_27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}). (\forall V4x \in A_27a. (p (ap (ap V0P V4x) (ap V3f V4x)))))))))) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)))))) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((\exists V1x \in A_27a. (p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0s))) \Leftrightarrow (\neg (V0s = (c_2Epred_set_2EEMPTY A_27a)))))) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EUNIV A_27a)))))) \quad (59)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V0s) V0s))) \quad (60)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) (ap (ap (c_2Epred_set_2EINTER A_27a) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t))))))) \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred_set.2EDIFF\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\\ & ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27a)\ V2x)\ V1t)))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EDIFF \\ & A.27a)\ V0s)\ V1t))\ V0s)))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealx.2Ereal.(p\ (ap\ (ap\ c.2Ereal.2Ereal_lte \\ & V0x)\ V0x))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty.2Erealx.2Ereal.(\forall V1b \in ty.2Erealx.2Ereal. \\ & ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal)\ V0a)\ (ap\ c.2Ereal_topology.2EClosed_segment \\ & (ap\ (ap\ (c.2Elist.2ECONS\ (ty.2Epair.2Eprod\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal))\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal)\ V0a)\ V1b))\ (c.2Elist.2ENIL\ (ty.2Epair.2Eprod \\ & ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal)))))) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & ty.2Erealx.2Ereal)\ V1b)\ (ap\ c.2Ereal_topology.2EClosed_segment \\ & (ap\ (ap\ (c.2Elist.2ECONS\ (ty.2Epair.2Eprod\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal))\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal)\ V0a)\ V1b))\ (c.2Elist.2ENIL\ (ty.2Epair.2Eprod \\ & ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal)))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty.2Erealx.2Ereal.(\forall V1b \in ty.2Erealx.2Ereal. \\ & (\forall V2x \in ty.2Erealx.2Ereal.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal) \\ & V2x)\ (ap\ c.2Ereal_topology.2EClosed_segment\ (ap\ (ap\ (c.2Elist.2ECONS \\ & (ty.2Epair.2Eprod\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal)) \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ & V0a)\ V1b))\ (c.2Elist.2ENIL\ (ty.2Epair.2Eprod\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal)))))) \Rightarrow ((p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ (\\ & ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\ & ty.2Erealx.2Ereal)\ V2x)\ V0a))\ (ap\ c.2Ereal_topology.2EDist \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ & V0a)\ V1b)))) \wedge (p\ (ap\ (ap\ c.2Ereal.2Ereal_lte\ (ap\ c.2Ereal_topology.2EDist \\ & (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\ & V2x)\ V1b))\ (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C \\ & ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal)\ V0a)\ V1b)))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\
& (p (ap c_2Ereal_topology_2Econnected (ap c_2Ereal_topology_2Eclosed_segment \\
& (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)) (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V0a) V1b)) (c_2Elist_2ENIL (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal)))))) \wedge (\forall V2a \in \\
& ty_2Erealax_2Ereal. (\forall V3b \in ty_2Erealax_2Ereal. (p (ap \\
& c_2Ereal_topology_2Econnected (ap c_2Ereal_topology_2Eopen_segment \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V2a) V3b)))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((ap c_2Ereal_topology_2Einterior \\
V0s) = (ap (ap (c_2Epred_set_2EDIFF ty_2Erealax_2Ereal) (c_2Epred_set_2EUNIV \\
ty_2Erealax_2Ereal)) (ap c_2Ereal_topology_2Eclosure (ap (\\
ap (c_2Epred_set_2EDIFF ty_2Erealax_2Ereal) (c_2Epred_set_2EUNIV \\
ty_2Erealax_2Ereal)) V0s))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V0s) \\
V1t)) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
(ap c_2Ereal_topology_2Eclosure V0s)) (ap c_2Ereal_topology_2Eclosure \\
V1t))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap c_2Ereal_topology_2Econnected V0s)) \wedge (\neg((ap (ap (c_2Epred_set_2EINTER \\
ty_2Erealax_2Ereal) V0s) V1t) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))) \wedge \\
& (\neg((ap (ap (c_2Epred_set_2EDIFF ty_2Erealax_2Ereal) V0s) V1t) = \\
(c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))))) \Rightarrow (\neg((ap (ap \\
(c_2Epred_set_2EINTER ty_2Erealax_2Ereal) V0s) (ap c_2Ereal_topology_2Efrontier \\
V1t)) = (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (p (ap (ap (c_2Epred_set_2ESUBSET \\
ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Efrontier (ap c_2Ereal_topology_2Einterior \\
V0s))) (ap c_2Ereal_topology_2Efrontier V0s))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2s_27 \in (2^{ty_2Erealax_2Ereal}). (\forall V3t_27 \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V2s_27) \\
& V0s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& V3t_27) V1t)) \wedge (\forall V4x \in ty_2Erealax_2Ereal. (\forall V5y \in \\
& ty_2Erealax_2Ereal. (((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V4x) V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5y) \\
& V1t)))) \Rightarrow (\exists V6x_27 \in ty_2Erealax_2Ereal. (\exists V7y_27 \in \\
& ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V6x_27) V2s_27)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V7y_27) V3t_27)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V6x_27) V7y_27)))) (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V4x) V5y)))))))))) \Rightarrow \\
& ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal} \\
& (2^{ty_2Erealax_2Ereal})) V2s_27) V3t_27)) = (ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) \\
& V0s) V1t)))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal} \\
& (2^{ty_2Erealax_2Ereal})) (ap c_2Ereal_topology_2Eclosure V0s)) \\
& V1t)) = (ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) V0s) V1t)))))) \wedge \\
& (\forall V2s \in (2^{ty_2Erealax_2Ereal}). (\forall V3t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal} \\
& (2^{ty_2Erealax_2Ereal})) V2s) (ap c_2Ereal_topology_2Eclosure \\
& V3t))) = (ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal} (2^{ty_2Erealax_2Ereal})) V2s) V3t))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Epred_set_2EDISJOINT ty_2Erealax_2Ereal) V0s) \\
& V1t)) \Rightarrow ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) (ap c_2Ereal_topology_2Efrontier \\
& V0s)) V1t)) = (ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V0s) V1t)))))) \wedge \\
& (\forall V2s \in (2^{ty_2Erealax_2Ereal}).(\forall V3t \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Epred_set_2EDISJOINT ty_2Erealax_2Ereal) V2s) \\
& V3t)) \Rightarrow ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V2s) (ap c_2Ereal_topology_2Efrontier \\
& V3t))) = (ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V2s) V3t)))))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((\neg(p (ap (ap (c_2Epred_set_2EDISJOINT ty_2Erealax_2Ereal) \\
& V0s) V1t))) \Rightarrow ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) V0s) V1t))) = \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{75}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{76}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee \neg(p \ V2r))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee \neg(p \ V0p)))))))) \\
& \tag{82}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))) \\
& \tag{83}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))) \\
& \tag{84}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p)))))) \\
& \tag{85}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \\
& \tag{86}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q)))) \\
& \tag{87}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap \ c_2Ereal_topology_2Esetdist \ (ap \ (ap \ (c_2Epair_2E_2C \ (2^{ty_2Erealax_2Ereal}) \\
& (2^{ty_2Erealax_2Ereal})) \ V0s) \ V1t)) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\
& ty_2Erealax_2Ereal) \ (ap \ (ap \ (c_2Epred_set_2EDISJOINT \ ty_2Erealax_2Ereal) \\
& V0s) \ V1t)) \ (ap \ c_2Ereal_topology_2Esetdist \ (ap \ (ap \ (c_2Epair_2E_2C \\
& (2^{ty_2Erealax_2Ereal}) \ (2^{ty_2Erealax_2Ereal})) \ (ap \ c_2Ereal_topology_2Efrontier \\
& V0s)) \ (ap \ c_2Ereal_topology_2Efrontier \ V1t)))) \ (ap \ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned}$$