

thm_2Ereal_topology_2ESETDIST__UNIV
(TMYSy9tcwZoiHpqdzDwQHWDWirQryr8UAQ8)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 10 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 11 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (5)$$

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Ereal_topology_2Esetdist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Esetdist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))}) \quad (7)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (9)$$

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (c_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal_lt\ T1\ T2)$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_2Ereal_2Ereal_lt\ x\ y)$

Definition 16 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ t1\ t2))\ (\lambda V2t \in 2.t))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 17 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (12)$$

Definition 18 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (16)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (20)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A_27a. (p (ap V1Q V3x))))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((\exists V1x \in A_27a. (p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0s))) \Leftrightarrow (\neg(V0s = (c_2Epred_set_2EEMPTY A_27a)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EUNIV A_27a)))) \quad (29)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal. ((ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealx_2Ereal ty_2Erealx_2Ereal) V0x) V0x)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (30)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. (((ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealx_2Ereal ty_2Erealx_2Ereal) V0x) V1y)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = V1y)))) \quad (31)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0x) V1y))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in (2^{ty_2Erealax_2Ereal}). ((ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)) V0t)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \wedge (\forall V1s \in (2^{ty_2Erealax_2Ereal}). ((ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& V1s) (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)))) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((ap c_2Ereal_topology_2Esetdist (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) \\
& (2^{ty_2Erealax_2Ereal})) V0s) V1t)) = (ap c_2Ereal_topology_2Esetdist \\
& (ap (ap (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& V1t) V0s))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2a \in ty_2Erealax_2Ereal. (\forall V3b \in ty_2Erealax_2Ereal. \\
& (\forall V4d \in ty_2Erealax_2Ereal. (((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V2a) V0s)) \wedge ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3b) V1t)) \wedge ((ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V2a) V3b)) = V4d) \wedge (\forall V5x \in \\
& ty_2Erealax_2Ereal. (\forall V6y \in ty_2Erealax_2Ereal. (((p (\\
& ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V0s)) \wedge (p (ap (ap \\
& (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6y) V1t))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V2a) V3b))) (ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V5x) V6y)))))))))) \Rightarrow ((ap c_2Ereal_topology_2Esetdist (ap (ap \\
& (c_2Epair_2E_2C (2^{ty_2Erealax_2Ereal}) (2^{ty_2Erealax_2Ereal})) \\
& V0s) V1t)) = V4d))))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned} & ((\forall V0s \in (2^{ty_2Erealax_2Ereal}).((ap\ c_2Ereal_topology_2Esetdist \\ & \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))) \\ V0s)\ (c_2Epred_set_2EUNIV\ ty_2Erealax_2Ereal)))) = (ap\ c_2Ereal_2Ereal_of_num \\ & \quad c_2Enum_2E0))) \wedge (\forall V1t \in (2^{ty_2Erealax_2Ereal}).((ap\ c_2Ereal_topology_2Esetdist \\ & \quad (ap\ (ap\ (c_2Epair_2E_2C\ (2^{ty_2Erealax_2Ereal})\ (2^{ty_2Erealax_2Ereal}))) \\ (c_2Epred_set_2EUNIV\ ty_2Erealax_2Ereal)\ V1t))) = (ap\ c_2Ereal_2Ereal_of_num \\ & \quad c_2Enum_2E0)))) \end{aligned}$$