

thm_2Ereal__topology_2ESPAN__BREAKDOWN
 (TMbdWrmfx-
 pEGSerPp2e6EFbDDwBrhvGL2km)

October 26, 2020

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P))))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2F)$.

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(3)

Definition 13 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 15 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 16 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap (ap c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(4)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(5)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(6)

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(7)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(8)

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num\ c_2Enum_2ESUC_REP)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
(9)

Definition 20 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B\ V0n) c_2Enum_2ESUC)$

Definition 21 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (12)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E.40\ t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (15)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Definition 25 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 26 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 27 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (18)$$

Definition 29 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) (19)$$

Definition 30 We define $c_2Ereal_topology_2Esubspace$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ c_2Ebock)$

Definition 31 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2Esubset))$

Definition 32 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_2EBigInter))$

Definition 33 We define $c_2Etopology_2Ehull$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).\lambda V1s \in (2^{A_27a}).(ap\ (c_2Etopology_2Ehull))$

Definition 34 We define $c_2Ereal_topology_2Espan$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ (c_2Etopology_2Espan)))$

Assume the following.

$$True (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) (22)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x))))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A_27a.(p (ap V1Q V3x))))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (36)$$

Assume the following.

$$(\forall V0r \in 2.(\forall V1p \in 2.(\forall V2q \in 2.(((p V1p) \wedge (p V2q)) \Rightarrow (p V0r)) \Leftrightarrow ((p V1p) \Rightarrow ((p V2q) \Rightarrow (p V0r)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0P)) \Leftrightarrow (p (ap V0P V1x)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1x \in A_27a.(\forall V2y \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) (ap (ap (c_2Epred_set_2EDELETE A_27a) V0s) V2y))) \Leftrightarrow ((p (ap (ap (c_2Ebool_2EIN A_27a) V1x) V0s)) \wedge (\neg(V1x = V2y))))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add V1y) V0x)))) \quad (40)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z)))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x)) \quad (42)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& \quad V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg \\
& \quad V0x)) (ap c_2Erealax_2Ereal_neg V1y))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V0x) V0x)))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Ereal_sub \\
& V0x) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& ((ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Ereal_sub \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y)) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0y) V1x))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((\forall V1a \in ty_2Erealax_2Ereal. \\
& (\forall V2s \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V1a) V2s)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V1a) (ap c_2Ereal_topology_2Espan V2s)))))) \wedge ((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& (ap c_2Ereal_topology_2Espan V0s)))) \wedge ((\forall V3x \in ty_2Erealax_2Ereal. \\
& (\forall V4y \in ty_2Erealax_2Ereal. (\forall V5s \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V3x) (ap c_2Ereal_topology_2Espan \\
& V5s))) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V4y) (ap \\
& c_2Ereal_topology_2Espan V5s)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) (ap (ap c_2Erealax_2Ereal_add V3x) V4y)) \\
& (ap c_2Ereal_topology_2Espan V5s)))))) \wedge ((\forall V6x \in ty_2Erealax_2Ereal. \\
& (\forall V7c \in ty_2Erealax_2Ereal. (\forall V8s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6x) (ap c_2Ereal_topology_2Espan \\
& V8s))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap (ap c_2Erealax_2Ereal_mul \\
& V7c) V6x)) (ap c_2Ereal_topology_2Espan V8s)))))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1h \in (2^{ty_2Erealax_2Ereal}). \\
& (((\forall V2x \in ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V2x) V0s)) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V2x) V1h)))) \wedge (p (ap c_2Ereal_topology_2Esubspace V1h))) \Rightarrow (\forall V3x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V3x) (ap c_2Ereal_topology_2Espan V0s))) \Rightarrow (p (ap V1h V3x)))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s)) \Rightarrow (p (\\
& ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) (ap c_2Ereal_topology_2Espan \\
& V0s))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2s \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_topology_2Espan V2s))) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_topology_2Espan \\
& V2s)))))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap (ap \\
& c_2Erealax_2Ereal_add V0x) V1y)) (ap c_2Ereal_topology_2Espan \\
& V2s))))))
\end{aligned} \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (80)$$

Theorem 1

$$\begin{aligned} & (\forall V0b \in ty_2Erealax_2Ereal. (\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\ & (\forall V2a \in ty_2Erealax_2Ereal. (((p (ap (ap (c_2Ebool_2EIN \\ ty_2Erealax_2Ereal) V0b) V1s)) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\ V2a) (ap c_2Ereal_topology_2Espan V1s)))) \Rightarrow (\exists V3k \in ty_2Erealax_2Ereal. \\ (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap (ap c_2Ereal_2Ereal_sub \\ V2a) (ap (ap c_2Erealax_2Ereal_mul V3k) V0b))) (ap c_2Ereal_topology_2Espan \\ (ap (ap (c_2Epred_set_2EDELETE ty_2Erealax_2Ereal) V1s) V0b)))))))))) \end{aligned}$$