

thm\_2Ereal\_\_topology\_2ESPAN\_\_BREAKDOWN\_\_EQ  
(TMUf-  
BUAFYLy7RsbNEHAWxE7Yfd8na9URc6Z)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 8** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 9** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A-27a}).(ap (c\_2Eenum\_2EZERO\_REP) : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EZERO\_REP \in \omega \tag{4}$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{5}$$

Let  $c\_2Eenum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EABS\_num \in (ty\_2Eenum\_2Eenum^{\omega}) \tag{6}$$

**Definition 10** We define  $c\_2Eenum\_2E0$  to be  $(ap\ c\_2Eenum\_2EABS\_num\ c\_2Eenum\_2EZERO\_REP)$ .

**Definition 11** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Eenum\_2E0$ .

Let  $c\_2Eenum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EREP\_num \in (\omega^{ty\_2Eenum\_2Eenum}) \tag{7}$$

Let  $c\_2Eenum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2ESUC\_REP \in (\omega^{\omega}) \tag{8}$$

**Definition 12** We define  $c\_2Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.(ap\ c\_2Eenum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Eenum\_2Eenum^{ty\_2Eenum\_2Eenum})^{ty\_2Eenum\_2Eenum}) \tag{9}$$

**Definition 13** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Eenum\_2Eenum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 14** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Eenum\_2Eenum.V0x$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{10}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{11}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{12}$$

**Definition 15** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the\ (\lambda x.x \in A.\lambda y.y \in A))$  of type  $\iota \Rightarrow \iota$ .

**Definition 16** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E.40 (t$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)) \quad (13)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etreall\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (14)$$

Let  $c\_2Erealax\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} \quad (16)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 19** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

**Definition 20** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E.21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 21** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E.3D\_3D\_3E V0t) c\_2Ebool\_2E$

**Definition 22** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 23** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap ($

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Eenum\_2Eenum} \quad (17)$$

Let  $c\_2Erealax\_2Etreall\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (18)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 25** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 26** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2E$

**Definition 27** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1x \in A\_27a.(ap (ap$   
 Let  $c\_2Erealax\_2Etreax\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) \quad (19)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 29** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 30** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E40$

**Definition 31** We define  $c\_2Ereal\_topology\_2Esubspace$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (ap c\_2Ebool$

**Definition 32** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap (c\_2Epred\_set$

**Definition 33** We define  $c\_2Etopology\_2Ehull$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).\lambda V1s \in (2^{A-27a}).(ap (c\_2$

**Definition 34** We define  $c\_2Ereal\_topology\_2Espan$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}).(ap (ap (c\_2Etopology$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\forall V1x \in \\
& A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\
& (2^{A.27a}).((\forall V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A.27a.(p \ ( \\
& ap \ V1Q \ V4x)))))) \quad (35)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))))) \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (43)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in \\ & A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).( \\ & \quad \forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ & A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ & V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\ & \quad V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1x \in \\ & A\_27a.(\forall V2y \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1x) \\ & (ap\ (ap\ (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V0s)\ V2y)))) \Leftrightarrow ((p\ (ap\ (ap \\ & \quad (c\_2Ebool\_2EIN\ A\_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & \quad V1y)\ V0x)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V1y)\ V2z)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & \quad (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y))\ V2z)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ c\_2Erealax\_2Ereal\_neg\ V0x))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & \quad c\_2Enum\_2E0))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & \quad V1y)\ V0x)))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\ & \quad ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ V0x) = V0x)) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x)) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Erealax\_2Ereal\_neg V0x)) (ap c\_2Erealax\_2Ereal\_neg V1y)))))) \quad (55)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V0x))) \quad (57)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V0x))) \Leftrightarrow (V0x = V1y)))) \quad (58)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)))))) \quad (59)$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte V0y) V1x)))))) \quad (60)$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}). (\forall V1t \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) V0s) \\
& V1t)) \Rightarrow (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& (ap c\_2Ereal\_topology\_2Espan V0s)) (ap c\_2Ereal\_topology\_2Espan \\
& V1t))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).((\forall V1a \in ty\_2Erealax\_2Ereal. \\
& (\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Ebool\_2EIN \\
ty\_2Erealax\_2Ereal) V1a) V2s)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
V1a) (ap c\_2Ereal\_topology\_2Espan V2s)))))) \wedge ((p (ap (ap (c\_2Ebool\_2EIN \\
ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
(ap c\_2Ereal\_topology\_2Espan V0s))) \wedge ((\forall V3x \in ty\_2Erealax\_2Ereal. \\
(\forall V4y \in ty\_2Erealax\_2Ereal. (\forall V5s \in (2^{ty\_2Erealax\_2Ereal}). \\
(((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V3x) (ap c\_2Ereal\_topology\_2Espan \\
V5s))) \wedge (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V4y) (ap \\
c\_2Ereal\_topology\_2Espan V5s)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN \\
ty\_2Erealax\_2Ereal) (ap (ap c\_2Erealax\_2Ereal\_add V3x) V4y)) \\
(ap c\_2Ereal\_topology\_2Espan V5s)))))) \wedge (\forall V6x \in ty\_2Erealax\_2Ereal. \\
(\forall V7c \in ty\_2Erealax\_2Ereal. (\forall V8s \in (2^{ty\_2Erealax\_2Ereal}). \\
((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V6x) (ap c\_2Ereal\_topology\_2Espan \\
V8s))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap (ap c\_2Erealax\_2Ereal\_mul \\
V7c) V6x)) (ap c\_2Ereal\_topology\_2Espan V8s)))))))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).(((p (ap (ap (c\_2Ebool\_2EIN \\
ty\_2Erealax\_2Ereal) V0x) (ap c\_2Ereal\_topology\_2Espan V2s)))) \wedge \\
(p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V1y) (ap c\_2Ereal\_topology\_2Espan \\
V2s)))))) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap (ap \\
c\_2Erealax\_2Ereal\_add V0x) V1y)) (ap c\_2Ereal\_topology\_2Espan \\
V2s))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty\_2Erealax\_2Ereal. (\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& (\forall V2a \in ty\_2Erealax\_2Ereal.(((p (ap (ap (c\_2Ebool\_2EIN \\
ty\_2Erealax\_2Ereal) V0b) V1s)) \wedge (p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
V2a) (ap c\_2Ereal\_topology\_2Espan V1s)))))) \Rightarrow (\exists V3k \in ty\_2Erealax\_2Ereal. \\
(p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap (ap c\_2Ereal\_2Ereal\_sub \\
V2a) (ap (ap c\_2Erealax\_2Ereal\_mul V3k) V0b))) (ap c\_2Ereal\_topology\_2Espan \\
(ap (ap (c\_2Epred\_set\_2EDELETE ty\_2Erealax\_2Ereal) V1s) V0b)))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (79)$$

### Theorem 1

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V0x) (ap c\_2Ereal\_topology\_2Espan (ap (ap (c\_2Epred\_set\_2EINSERT ty\_2Erealax\_2Ereal) V1a) V2s)))) \Leftrightarrow (\exists V3k \in ty\_2Erealax\_2Ereal.(p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap (ap c\_2Ereal\_2Ereal\_sub V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V3k) V1a))) (ap c\_2Ereal\_topology\_2Espan V2s))))))))))$$