

thm_2Ereal_topology_2ESPAN_TRANS (TMJ7yYjztibGqRjSpUde7LgNHj1BmChUmSi)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(\lambda V1f \in (2^{A_27a}).(\lambda V1f V0P)))$

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 9 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27a))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{7}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{8}$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{9}$$

Definition 13 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))\ V0n)$

Definition 14 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{10}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{11}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{12}$$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 16 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ a))\ V0a)$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 17 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (14)$$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (17)$$

Definition 18 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$.

Definition 19 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$.

Definition 20 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 21 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$.

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 23 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ ($

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 24 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 25 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 26 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2E$

Definition 27 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap\ (ap$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (19)$$

Definition 28 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 29 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 30 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 31 We define $c_2Ereal_topology_2Esubspace$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ c_2Ebool$

Definition 32 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap\ (c_2Epred_s$

Definition 33 We define $c_2Etopology_2EHull$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).\lambda V1s \in (2^{A-27a}).(ap\ (c_2$

Definition 34 We define $c_2Ereal_topology_2Espan$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (ap\ (c_2Etopology$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge (p\ V1t2) \wedge (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (27)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A.27a. (p\ (ap\ V1Q\ V3x))))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (40)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}). (\forall V4x \in A.27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. (\forall V2s \in (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (ap\ (c.2Epred._set.2EINSERT\ A.27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ V2s)))))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1x \in A.27a. (\forall V2y \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ (ap\ (ap\ (c.2Epred._set.2EDELETE\ A.27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add V1y) V0x)))) \quad (45)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z)))))) \quad (46)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (47)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x)) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x)) \quad (49)$$

Assume the following.

$$(V0x) (ap c_2Erealax_2Ereal_neg V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y)))))) \quad (51)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)))))) \quad (55)$$

Assume the following.

$$(\forall V0y \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte V0y) V1x)))))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y)))))) \quad (57)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))))) \quad (58)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg (ap c_2Erealax_2Ereal_neg V0x)) = V0x)) \quad (59)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))))) \quad (60)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V0s) \\
& V1t)) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap c_2Ereal_topology_2Espan V0s)) (ap c_2Ereal_topology_2Espan \\
& V1t))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2s \in (2^{ty_2Erealax_2Ereal}). (((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_topology_2Espan V2s))) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_topology_2Espan \\
& V2s)))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap (ap \\
& c_2Erealax_2Ereal_add V0x) V1y)) (ap c_2Ereal_topology_2Espan \\
& V2s))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1c \in ty_2Erealax_2Ereal. \\
& (\forall V2s \in (2^{ty_2Erealax_2Ereal}). ((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_topology_2Espan V2s))) \Rightarrow \\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap (ap c_2Erealax_2Ereal_mul \\
& V1c) V0x)) (ap c_2Ereal_topology_2Espan V2s))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Erealax_2Ereal. (\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V2a \in ty_2Erealax_2Ereal. (((p (ap (ap (c_2Ebool_2EIN \\
& ty_2Erealax_2Ereal) V0b) V1s)) \wedge (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V2a) (ap c_2Ereal_topology_2Espan V1s)))) \Rightarrow (\exists V3k \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap (ap c_2Ereal_2Ereal_sub \\
& V2a) (ap (ap c_2Erealax_2Ereal_mul V3k) V0b))) (ap c_2Ereal_topology_2Espan \\
& (ap (ap (c_2Epred_set_2EDELETE ty_2Erealax_2Ereal) V1s) V0b))))))
\end{aligned} \tag{65}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{66}$$

Assume the following.

$$(\forall V0A \in 2.((p \vee 0A) \Rightarrow ((\neg(p \vee 0A)) \Rightarrow \text{False}))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \vee 0A) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \vee 0A)) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \vee 0A)) \Rightarrow \text{False}) \Rightarrow (((p \vee 0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Leftrightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((\neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))) \quad (75)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2s \in (2^{ty_2Erealax_2Ereal}). (((p (ap (ap (c_2Ebool_2EIN \\ & ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_topology_2Espan V2s))) \wedge \\ & (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_topology_2Espan \\ & (ap (ap (c_2Epred_set_2EINSERT ty_2Erealax_2Ereal) V0x) V2s)))))) \Rightarrow \\ & (p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1y) (ap c_2Ereal_topology_2Espan \\ & V2s)))))) \end{aligned}$$