

thm_2Ereal__topology_2ESPHERE__TRANSLATION (TMWs4Q8c97WEiVBuqJ8U8t1mJGYbGKUCQzf)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p (ap P x))$) of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 5 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P) (c_2Emin_2E_3D (2^{A_27a}) P))))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Emin_2E_3D (2^{A_27a}) (ap (c_2Emin_2E_3D (2^{A_27b}) V1y) V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(3)

Definition 10 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(4)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(5)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(6)

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 12 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(7)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(8)

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
(9)

Definition 14 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 15 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
(10)

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal$$
(11)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax})$$
(12)

Definition 16 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$
Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (13)$$

Definition 17 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$
Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (14)$$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (16)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 18 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 19 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 20 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 22 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (18)$$

Definition 23 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreall_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (19)$$

Definition 24 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 25 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 27 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)} \quad (20)$$

Let $c_2Ereal_topology_2Esphere : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Esphere \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)}) \quad (21)$$

Definition 28 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (32)$$

Assume the following.

$$2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V0x)))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
(ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
(ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg \\
V0x)) (ap c_2Erealax_2Ereal_neg V1y))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
(ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
V0x) V0x)))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\ V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (45)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ ((ap (ap c_2Erealax_2Ereal_add V1y) (ap (ap c_2Ereal_2Ereal_sub \\ V0x) V1y)) = V0x))) \quad (46)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ ((ap (ap c_2Ereal_2Ereal_sub (ap (ap c_2Erealax_2Ereal_add \\ V0x) V1y)) V0x) = V1y))) \quad (47)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\ V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\ V0x) V1y)))))) \quad (48)$$

Assume the following.

$$(\forall V0y \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal. \\ ((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\ V0y) V1x))))) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\ V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\ (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\ V1y) V0x))))) \quad (51)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg \\ (ap c_2Erealax_2Ereal_neg V0x)) = V0x)) \quad (52)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y)))) \Leftrightarrow (p (ap (ap (c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_topology_2EDist (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal) V0x) V1y)) = (ap c_2Ereal_2Eabs (ap (ap c_2Ereal_2Ereal_sub \\
& V0x) V1y))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2e \in ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& V1y) (ap c_2Ereal_topology_2Esphere (ap (ap (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0x) V2e)))) \Leftrightarrow ((ap c_2Ereal_topology_2EDist \\
& (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0x) V1y)) = V2e))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (64)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (69)$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal. \\ & (\forall V2r \in ty_2Erealax_2Ereal.((ap\ c_2Ereal_topology_2Esphere \\ & (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\ & (ap\ (ap\ c_2Erealax_2Ereal_add\ V0a)\ V1x))\ V2r)) = (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) (\lambda V3y \in ty_2Erealax_2Ereal. \\ & (ap\ (ap\ c_2Erealax_2Ereal_add\ V0a)\ V3y))\ (ap\ c_2Ereal_topology_2Esphere \\ & (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\ & V1x)\ V2r)))))) \end{aligned}$$