# thm_2Ereal__topology_2ESUMMABLE__BILINEAR__PARTIAL__P (TMUptEFsfp8FqR65f92V884eYU89dxi8cAW)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 3** *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

**Definition 4** *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

**Definition 5** *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

**Definition 7** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a})).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

**Definition 8** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (6)$$

**Definition 9** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

**Definition 10** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (8)$$

**Definition 11** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

**Definition 12** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 13** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (10)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (11)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (12)$$

**Definition 14** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$
*of type* $\iota \Rightarrow \iota$.

**Definition 15** *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{13}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{14}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{15}$$

**Definition 16** *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

**Definition 17** *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{16}$$

**Definition 18** *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

**Definition 19** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 20** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

**Definition 21** *We define* c_2Ereal__topology_2Elinear *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$

**Definition 22** *We define* c_2Ereal__topology_2Ebilinear *to be* $\lambda V0f \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ere}$

**Definition 23** *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

**Definition 24** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

**Definition 25** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

**Definition 26** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 27** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

**Definition 28** *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{17}$$

**Definition 29** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (18)$$

**Definition 30** *We define* c_2Ereal__topology_2Efrom *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Epred\_\_set\_$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

**Definition 31** *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

**Definition 32** *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $ty\_2Ereal\_\_topology\_2Enet : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Ereal\_\_topology\_2Enet\ A0) \quad (20)$$

**Definition 33** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 34** *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 35** *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Ereal\_\_topology\_2Emk\_\_net : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ereal\_\_topology\_2Emk\_\_net\ A\_27a \in ((ty\_2Ereal\_\_topology\_2Enet\ A\_27a)^{((2^{A\_27a})^{A\_27a})}) \quad (21)$$

**Definition 36** *We define* c_2Ereal__topology_2Esequentially *to be* $(ap\ (c\_2Ereal\_\_topology\_2Emk\_\_net\ ty\_2$

**Definition 37** *We define* c_2Eiterate_2E_2E_2E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 38** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

**Definition 39** *We define* c_2Epred__set_2EINTER *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

**Definition 40** *We define* c_2Eiterate_2Eneutral *to be* $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap$ (c_2Emin

**Definition 41** *We define* c_2Eiterate_2Esupport *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 42** *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c$

**Definition 43** *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.$c_2Ebool_2EF$)$.

**Definition 44** *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

**Definition 45** *We define* c_2Eiterate_2EITSET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V$

**Definition 46** *We define* c_2Eiterate_2Eiterate *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V$

**Definition 47** *We define* c_2Eiterate_2ESum *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Eiterate\_2Eiterate}\ A\_27a\ ty\_2Erealax$

Let $c\_2Ereal\_\_topology\_2EDist : \iota$ be given. Assume the following.

$$c\_2Ereal\_\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \tag{22}$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{23}$$

**Definition 48** *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Ereal\_\_topology\_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ereal\_\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_\_topology\_2Enet\ A\_27a)}) \tag{24}$$

**Definition 49** *We define* c_2Ereal__topology_2Etrivial__limit *to be* $\lambda A\_27a : \iota.\lambda V0net \in (ty\_2Ereal\_\_topolog$

**Definition 50** *We define* c_2Ereal__topology_2Eeventually *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1net \in (ty\_2$

**Definition 51** *We define* c_2Ereal__topology_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Erealax\_2Ereal^{A}$

**Definition 52** *We define* c_2Ereal__topology_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda$

**Definition 53** *We define* c_2Ereal__topology_2Esummable *to be* $\lambda V0s \in (2^{ty\_2Enum\_2Enum}).\lambda V1f \in (ty\_2E$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \tag{31}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{35}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{36}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27b)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27b))\ V0f) = V0f) \land ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)) = V0f))) \tag{39}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V2h \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{ty\_2Erealax\_2Ereal} (\forall V3m \in ty\_2Enum\_2Enum.(\forall V4n \in ty\_2Enum\_2Enum.( (p\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Ebilinear}\ V2h)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Eiterate\_2ESum}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ V3m)\ V4n))\ (\lambda V5k \in ty\_2Enum\_2Enum.(ap\ (ap\ V2h\ (ap\ V0f\ V5k))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V1g\ V5k))\ (ap\ V1g\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V5k)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))) = ( ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V3m)\ V4n))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ V2h\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V4n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (ap\ V1g\ V4n)))\ (ap\ (ap\ V2h\ (ap\ V0f\ V3m))\ (ap\ V1g\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V3m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))\ (ap\ (ap\ (\mathsf{c\_2Eiterate\_2ESum}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ V3m)\ V4n))\ (\lambda V6k \in ty\_2Enum\_2Enum. (ap\ (ap\ V2h\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6k)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) (ap\ V0f\ V6k)))\ (ap\ V1g\ V6k))))))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))))))) \tag{40}$$

Assume the following.

$$(\forall V0k \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINTER}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Ereal\_\_topology\_2Efrom}\ V0k))\ (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ \mathsf{c\_2Enum\_2E0})\ V1n)) = (ap\ (ap\ \mathsf{c\_2Eiterate\_2E\_2E\_2E}\ V0k)\ V1n)))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0net \in (ty\_2Ereal\_\_topology\_2Enet\ A\_27a).(\forall V1a \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ A\_27a)\ (\lambda V2x \in A\_27a.V1a))\ V1a)\ V0net)))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0net \in (ty\_2Ereal\_\_topology\_2Enet\ A\_27a).(\forall V1f \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V2g \in (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V3l \in ty\_2Erealax\_2Ereal.(\forall V4m \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ A\_27a)\ V1f)\ V3l)\ V0net))\wedge(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ A\_27a)\ V2g)\ V4m)\ V0net)))\Rightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ A\_27a)\ (\lambda V5x \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V1f\ V5x))\ (ap\ V2g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V3l)\ V4m))\ V0net))))))) \tag{43}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3m \in ty\_2Enum\_2Enum.((((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ (\lambda V4n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V3m)\ V4n))\ (ap\ V0f\ V4n))\ (ap\ V1g\ V4n))))\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially}))\Leftrightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ V0f)\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially})))\wedge(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ (\lambda V5n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3m)\ V5n))\ (ap\ V0f\ V5n))\ (ap\ V1g\ V5n))))\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially}))\Leftrightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ V0f)\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially})))\wedge(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ (\lambda V6n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V6n)\ V3m))\ (ap\ V0f\ V6n))\ (ap\ V1g\ V6n))))\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially}))\Leftrightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ V1g)\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially})))\wedge((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ (\lambda V7n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V7n)\ V3m))\ (ap\ V0f\ V7n))\ (ap\ V1g\ V7n))))\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially}))\Leftrightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ereal\_\_topology\_2E\_2D\_2D\_3E}\ ty\_2Enum\_2Enum)\ V1g)\ V2l)\ \mathsf{c\_2Ereal\_\_topology\_2Esequentially}))))))))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{49}$$

**Theorem 1**

$$\begin{gathered}
(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\
(ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V2h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal} \\
(\forall V3l \in ty_2Erealax_2Ereal.(\forall V4k \in ty_2Enum_2Enum. \\
(((p\ (ap\ \mathsf{c_2Ereal__topology_2Ebilinear}\ V2h)) \wedge ((p\ (ap\ (ap\ (ap\ ( \\
\mathsf{c_2Ereal__topology_2E_2D_2D_3E}\ ty_2Enum_2Enum)\ (\lambda V5n \in ty_2Enum_2Enum. \\
(ap\ (ap\ V2h\ (ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V5n)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ V1g \\
V5n))))\ V3l)\ \mathsf{c_2Ereal__topology_2Esequentially})) \wedge (p\ (ap\ (ap \\
\mathsf{c_2Ereal__topology_2Esummable}\ (ap\ \mathsf{c_2Ereal__topology_2Efrom} \\
V4k))\ (\lambda V6n \in ty_2Enum_2Enum.(ap\ (ap\ V2h\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub} \\
(ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V6n)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))))\ (ap\ V0f \\
V6n)))\ (ap\ V1g\ V6n))))))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c_2Ereal__topology_2Esummable} \\
(ap\ \mathsf{c_2Ereal__topology_2Efrom}\ V4k))\ (\lambda V7n \in ty_2Enum_2Enum. \\
(ap\ (ap\ V2h\ (ap\ V0f\ V7n))\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub}\ (ap\ V1g\ V7n)) \\
(ap\ V1g\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V7n)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL} \\
(ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO}))))))))))))))
\end{gathered}$$