

thm_2Ereal__topology_2ESUMMABLE__IMP__SUMS__BOUNDED (TMdipEqm6ZWxV6LhbGgeSicpnC1diPMSSJw)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(3)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(4)

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(5)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(6)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(7)

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Ereal_topology_2Efrom$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Epred_set_2EGSPEC$

Definition 17 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 18 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
(8)

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal$$
(9)

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal})$$
(10)

Definition 19 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (11)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (12)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (13)$$

Definition 20 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 21 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (14)$$

Definition 22 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (15)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (16)$$

Definition 23 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 24 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 26 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Definition 27 We define $c_2Ereal_topology_2Ebounded_def$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebo$

Definition 28 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 29 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ereal_topology_2Enet\ A0) \quad (17)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A.27a \in ((ty_2Ereal_topology_2Enet\ A.27a)^{(2^{A-27a})^{A-27a}}) \end{aligned} \quad (18)$$

Definition 30 We define $c_2Ereal_topology_2Esequentially$ to be $(ap\ (c_2Ereal_topology_2Emk_net\ ty_2Ereal_topology_2Esequentially))$.

Definition 31 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 32 We define $c_2Epred_set_2EINTER$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2Ereal_topology_2Einter\ A.27a\ V0s\ V1t))$.

Let $c_2Erealax_2Etreax_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 33 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 34 We define $c_2Eiterate_2Eneutral$ to be $\lambda A.27a : \iota.\lambda V0op \in ((A.27a^{A-27a})^{A-27a}).(ap\ (c_2Eiterate_2Eneutral\ A.27a\ V0op))$.

Definition 35 We define $c_2Eiterate_2Esupport$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0op \in ((A.27b^{A-27b})^{A-27b}).\lambda V1op \in ((A.27a^{A-27a})^{A-27a}).(ap\ (c_2Eiterate_2Esupport\ A.27a\ A.27b\ V0op\ V1op))$.

Definition 36 We define $c_2Epred_set_2EINSERT$ to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A-27a}).(ap\ (c_2Epred_set_2Einsert\ A.27a\ V0x\ V1s))$.

Definition 37 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2EF)$.

Definition 38 We define $c_2Epred_set_2EFINITE$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).(ap\ (c_2Ebool_2E21\ A.27a\ V0s))$.

Definition 39 We define $c_2Eiterate_2EITSET$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in ((A.27a^{A-27a})^{A-27b}).\lambda V1f \in ((A.27b^{A-27b})^{A-27a}).(ap\ (c_2Eiterate_2Eitset\ A.27a\ A.27b\ V0f\ V1f))$.

Definition 40 We define $c_2Eiterate_2Eiterate$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0op \in ((A.27b^{A-27b})^{A-27b}).\lambda V1op \in ((A.27a^{A-27a})^{A-27a}).(ap\ (c_2Eiterate_2Eiterate\ A.27a\ A.27b\ V0op\ V1op))$.

Definition 41 We define $c_2Eiterate_2ESum$ to be $\lambda A.27a : \iota.(ap\ (c_2Eiterate_2Eiterate\ A.27a\ ty_2Erealax_2Ereal_add\ A.27a))$.

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (20)$$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ereal_topology_2Enetord\ A.27a \in (((2^{A-27a})^{A-27a})^{(ty_2Ereal_topology_2Enet\ A.27a)}) \quad (21)$$

Definition 42 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A.27a : \iota.\lambda V0net \in (ty_2Ereal_topology_2Etrivial_limit\ A.27a)$.

Definition 43 We define $c_Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1net \in (ty_2E$

Definition 44 We define $c_Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal^A$

Definition 45 We define $c_Ereal_topology_2Esums$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda$

Definition 46 We define $c_Ereal_topology_2Esummable$ to be $\lambda V0s \in (2^{ty_2Enum_2Enum}).\lambda V1f \in (ty_2E$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow & (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow & (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ & 2.(((\exists V2x \in A_27a.(p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in \\ & A_27a.((p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0k \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap (ap (c_2Epred_set_2EINTER ty_2Enum_2Enum) (ap c_2Ereal_topology_2Efrom \\ & V0k)) (ap (ap c_2Eiterate_2E_2E_2E c_2Enum_2E0) V1n)) = (ap (ap \\ & c_2Eiterate_2E_2E_2E V0k) V1n)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1l \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (ap (c_2Ereal_topology_2E_2D_2D_3E \\
& ty_2Enum_2Enum) V0s) V1l) c_2Ereal_topology_2Esequentially)) \Rightarrow \\
& (p (ap c_2Ereal_topology_2Ebounded_def (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Enum_2Enum ty_2Erealax_2Ereal) V0s) (c_2Epred_set_2EUNIV \\
& ty_2Enum_2Enum)))))))))
\end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1k \in \\
& ty_2Enum_2Enum.((p (ap (ap c_2Ereal_topology_2Esummable (ap \\
& c_2Ereal_topology_2Efrom V1k)) V0f)) \Rightarrow (p (ap c_2Ereal_topology_2Ebounded_def \\
& (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Enum_2Enum) \\
& (\lambda V2n \in ty_2Enum_2Enum.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& 2) (ap (ap (c_2Eiterate_2ESum ty_2Enum_2Enum) (ap (ap c_2Eiterate_2E_2E_2E \\
& V1k) V2n)) V0f)) (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V2n) (c_2Epred_set_2EUNIV \\
& ty_2Enum_2Enum)))))))))
\end{aligned}$$